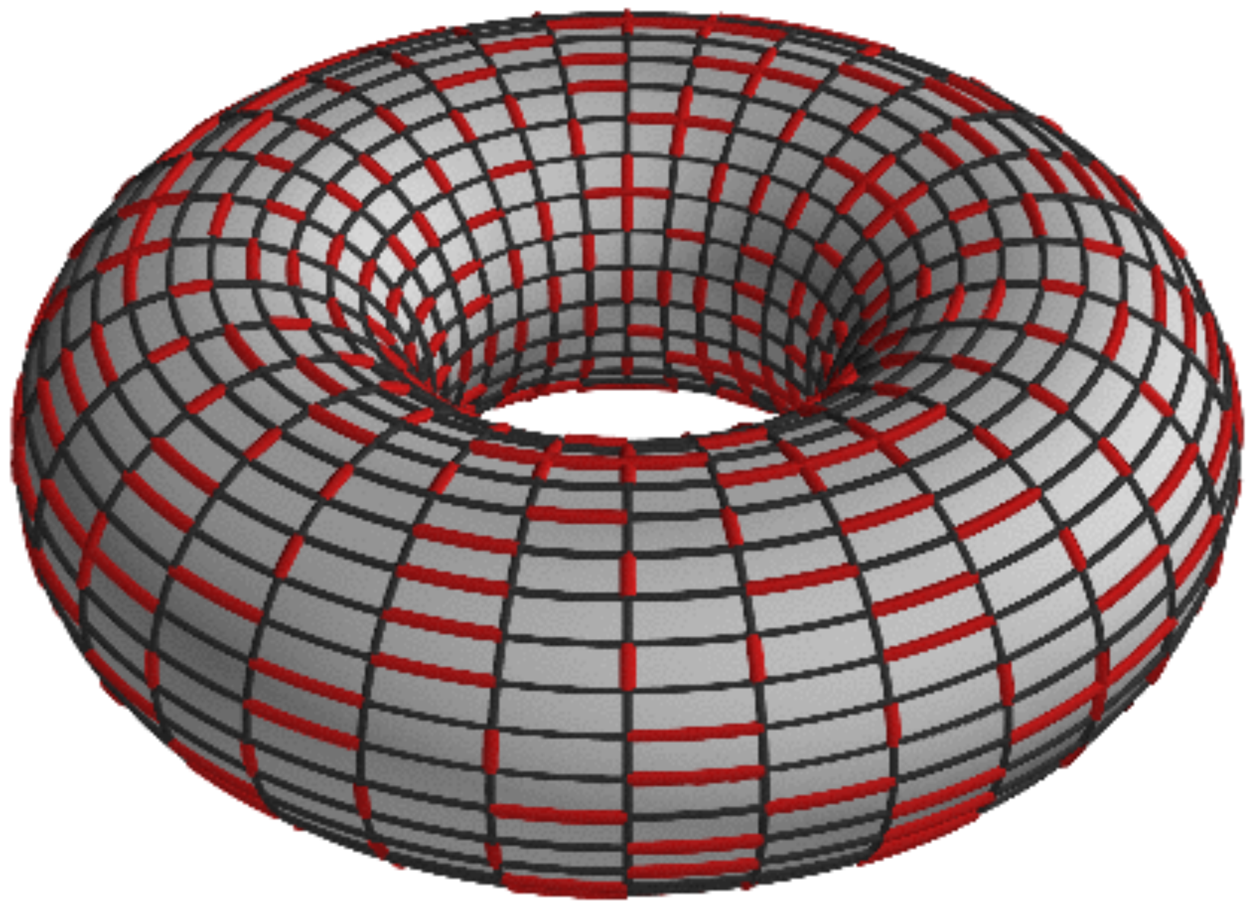
# A Numerical Study of the Energy Gap of the Quantum Dimer-Pentamer Model



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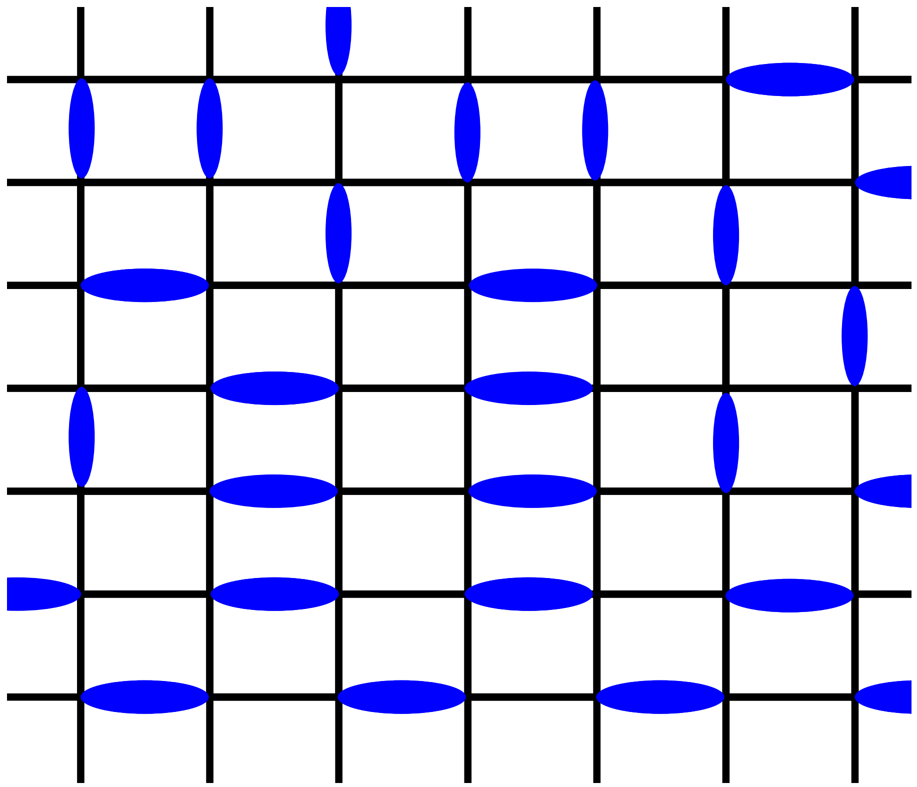
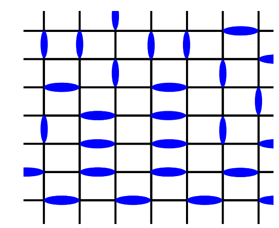
# Quantum Dimer Model



Hilbert space ➔ $\{C\}$

$$C_n = $$ 

$$H = -t\sum_{\square}\left(|\,\|\,\|\,\rangle\langle\,=\,| + \text{h.c.}\right)$$

$$+\,v\sum_{\square}\left(|\,\|\,\|\,\rangle\langle\,\|\,\|\,| + |\,=\,\rangle\langle\,=\,|\right)$$

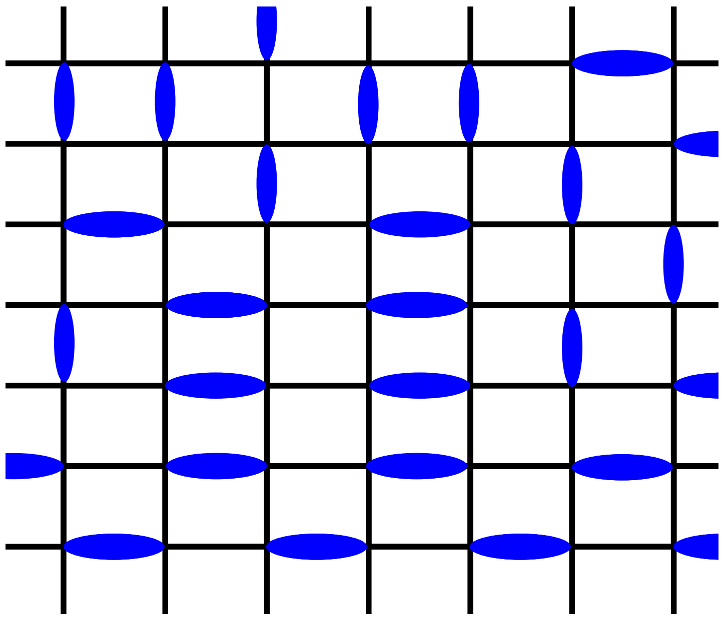
RK point

$$t/v = 1$$

$$|\Psi_{\mathrm{RK}}\rangle = \frac{1}{\sqrt{\mathcal{N}}}\sum_{C}|C\rangle$$

**Rokhsar, Kivelson, Phys. Rev. Lett. 1988.**

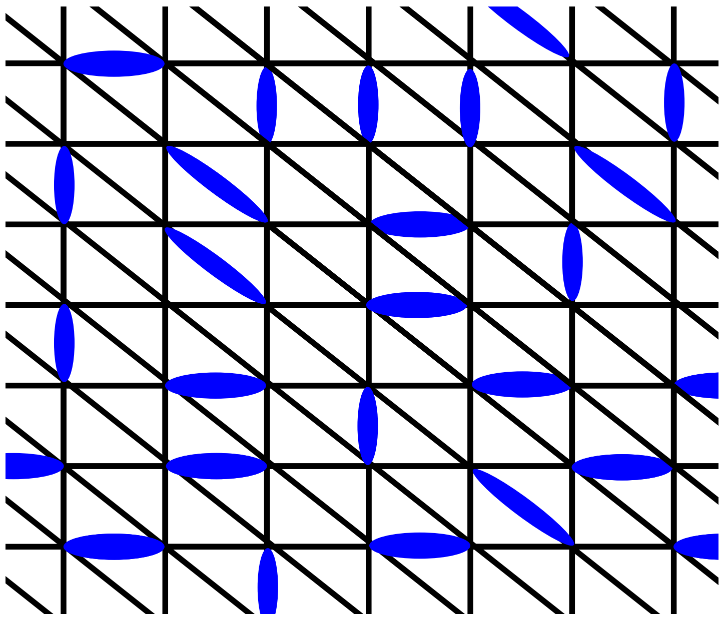
## Square Lattice Quantum Dimer Model (QDM)



(At the RK Point)

- Gapless
- Power law decay of dimer correlations in liquid state
- Extensive topological degeneracy

## Triangular Lattice QDM

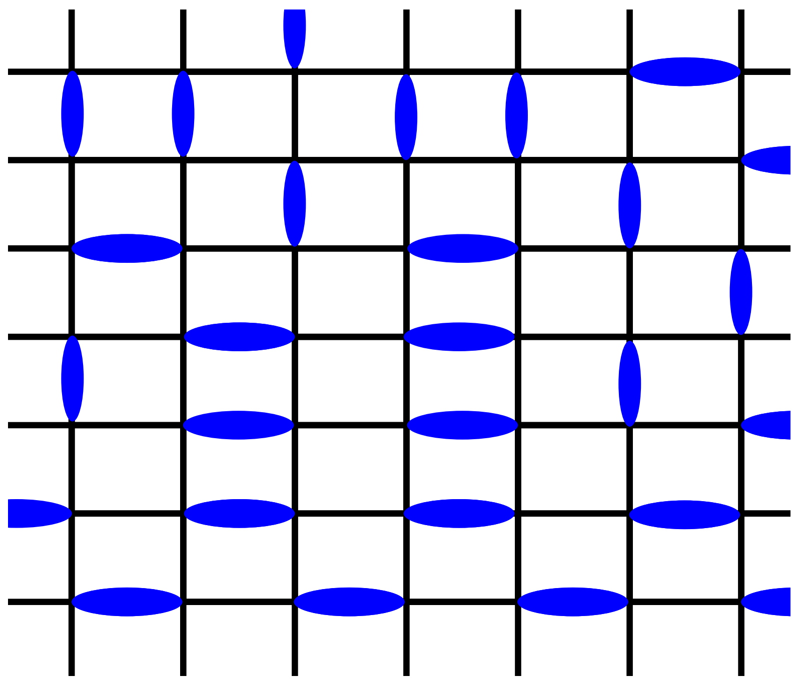


(At the RK Point)

- Gapped
- Exponential decay of dimer correlations in liquid state
- Finite topological degeneracy
- $Z_2$ topological order

**Rokhsar, Kivelson, Phys. Rev. Lett. 1988.**
**Moessner, Sondhi, Phys. Rev. Lett. 2001.**

# Square Lattice Quantum Dimer Model



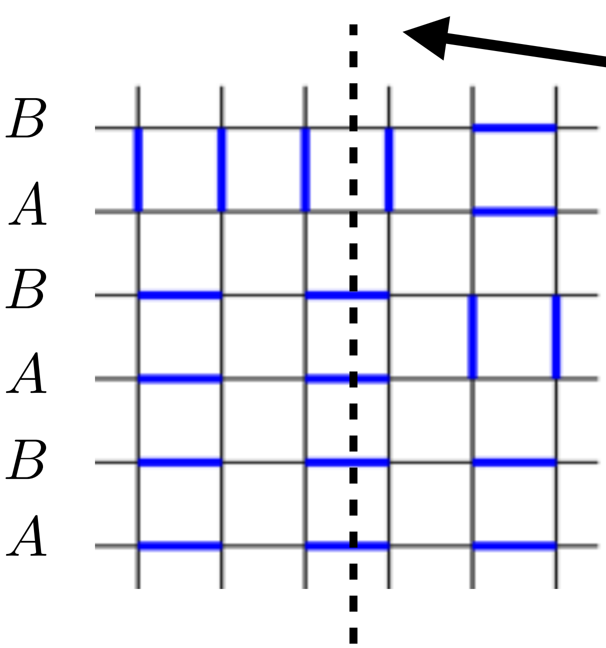
## Local Constraints

One dimer touching each vertex

$$e^{i\alpha(n_v-1)}|\psi\rangle = |\psi\rangle$$

For $n_v = 1$ $\alpha$ is 0 - $2\pi$

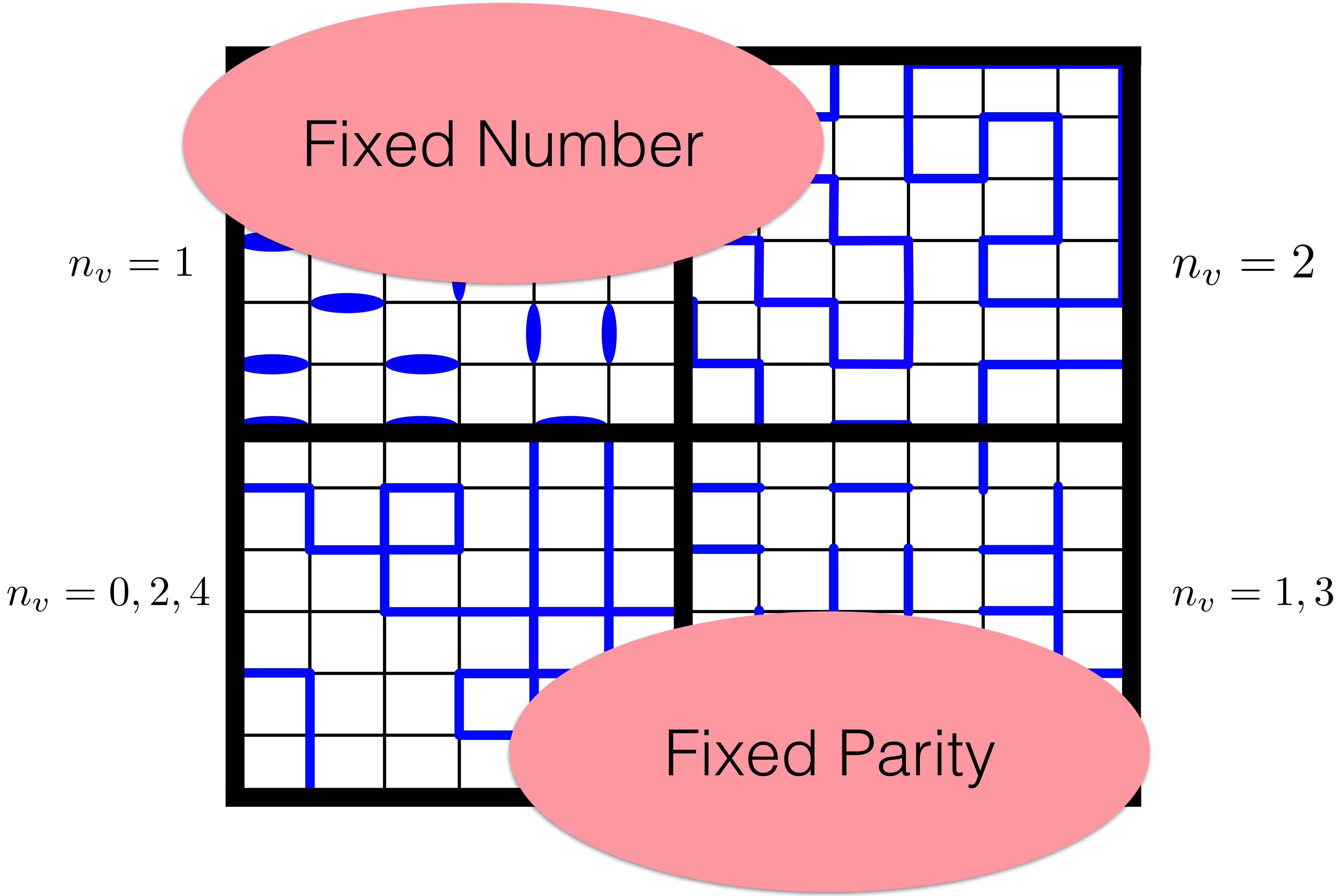
$U(1)$ Local gauge symmetry




$$W^y_{QDM} = N_A - N_B = \begin{cases} -L/2 \\ \vdots \\ -1 \\ 0 \\ 1 \\ \vdots \\ L/2 \end{cases}$$

$$[H_{QDM}, W^y_{QDM}] = 0$$

**Moessner, Sondhi, Fradkin, Phys. Rev. B. 2001**



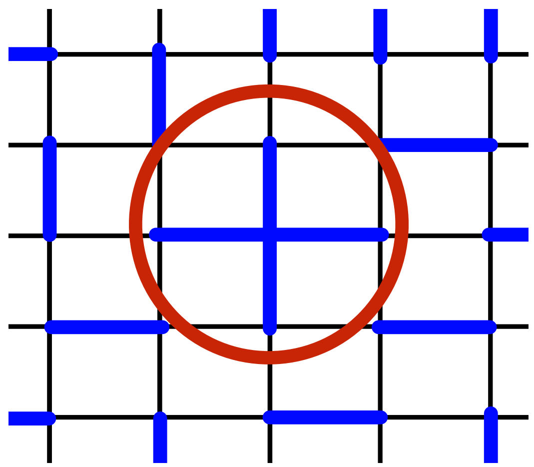
Fixed Number
$n_v = 1$
$n_v = 2$
$n_v = 0, 2, 4$
$n_v = 1, 3$
Fixed Parity

# Possibilities on the square lattice

| Dimers at vertex | Corresponding Model | Exact Local Gauge Symmetry | |
|---|---|---|---|
| 0 (4) | Trivial Case | | |
| 1 (3) | QDM | $U(1)$ | gapless, extensive topological degen. |
| 2 | Fully Packed Loop Model | $U(1)$ | gapless, extensive topological degen. |
| 1, 3 | Toric Code Odd Parity | $Z_2$ | gapped, finite topological degen. |
| 0, 2, 4 | Toric Code Even Parity | $Z_2$ | gapped, finite topological degen. |

What else is possible? → Quantum Dimer Pentamer Model (QDPM)

# Quantum Dimer Pentamer Model (QDPM)

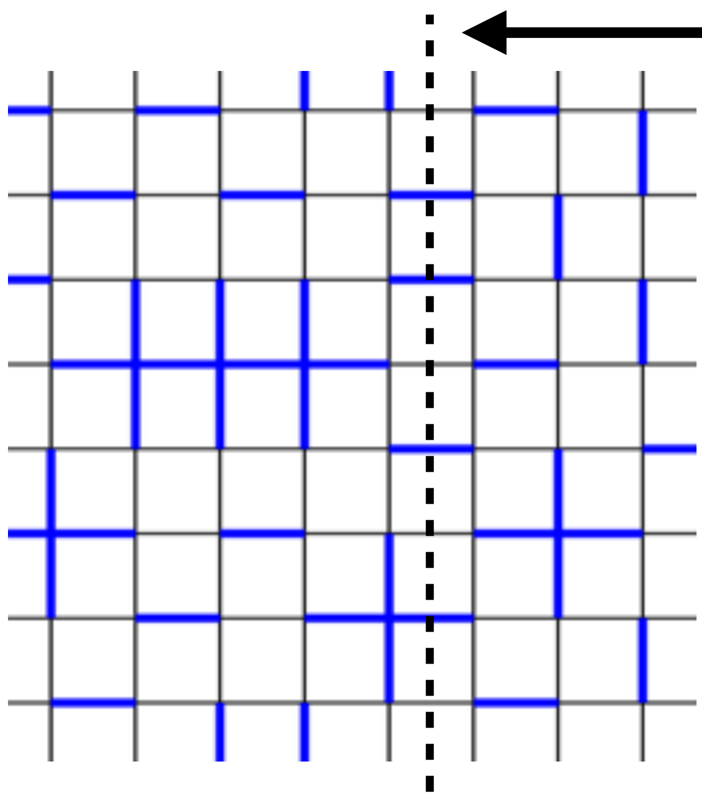


$n_v = 1,\ 4$

$e^{i\alpha(n_v - 1)}$ $\quad \alpha = \{0, 2\pi/3, -2\pi/3\}$

$Z_3$ Local gauge symmetry

<u>In the QDPM a new winding number is conserved</u>



$$W^y_{QDPM} = (N_A - N_B)\text{mod}3 = \begin{cases} 0 \\ 1 \\ 2 \end{cases}$$

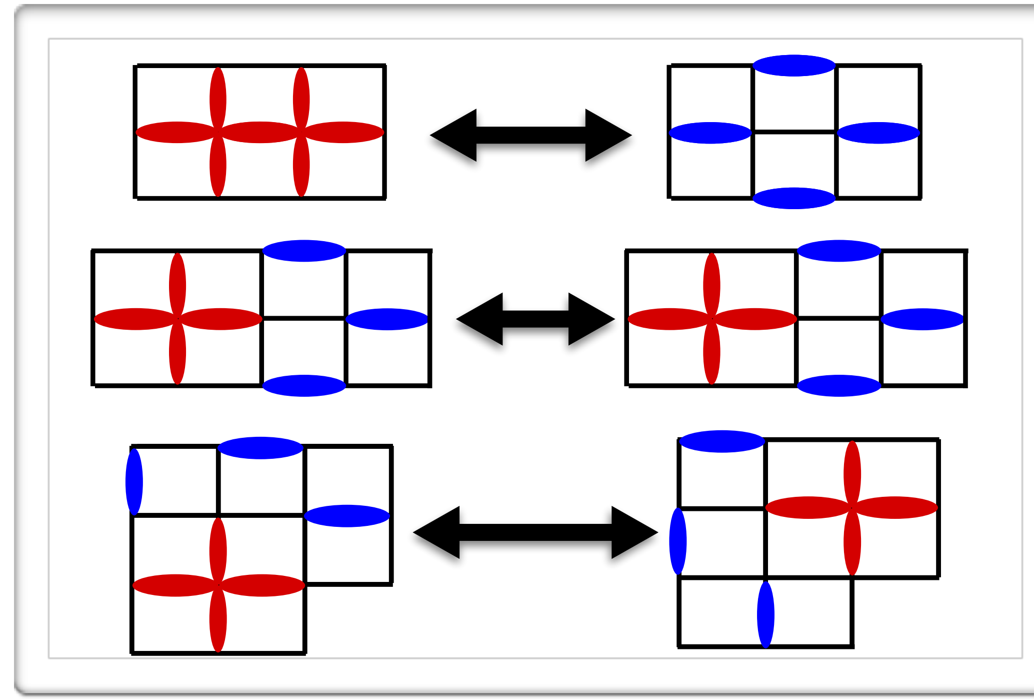
$$[H_{QDPM}, W^y_{QDM}] \neq 0$$

$$[H_{QDPM}, W^y_{QDPM}] = 0$$

3x3 fold topological degeneracy on a torus.

Extensive $\longrightarrow$ Finite top. degeneracy

# Pentamer Dynamics and Hamiltonian



$$H_{\mathrm{QDPM}} = H_{\mathrm{QDM}} + H_{\mathrm{pent.}}$$

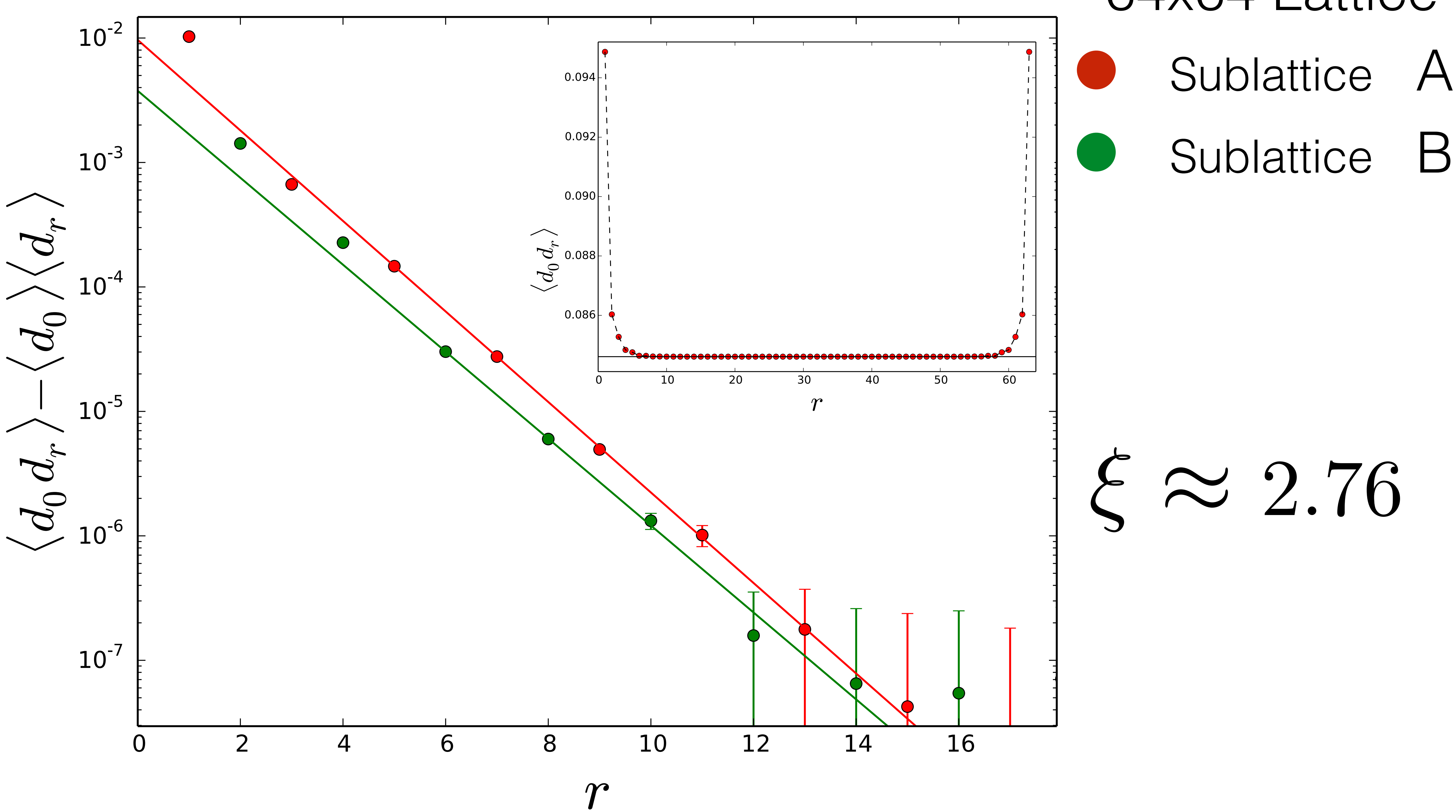
$$H_{\mathrm{pent.}} = \sum_i \Big(\text{-}t_i \sum \mathrm{KE}_i + v_i \sum \mathrm{PE}_i\Big)$$

$$|\Psi_{\mathrm{RK}}\rangle = \frac{1}{\sqrt{\mathcal{N}}} \sum_C |C\rangle$$

RK point

$$t_i/v_i = 1$$

- Using Monte Carlo method we sample the ground state wave function.
- Power law dimer correlations in QDM $\rightarrow$ QDPM has exponential dimer correlations.
- No evidence of symmetry breaking.



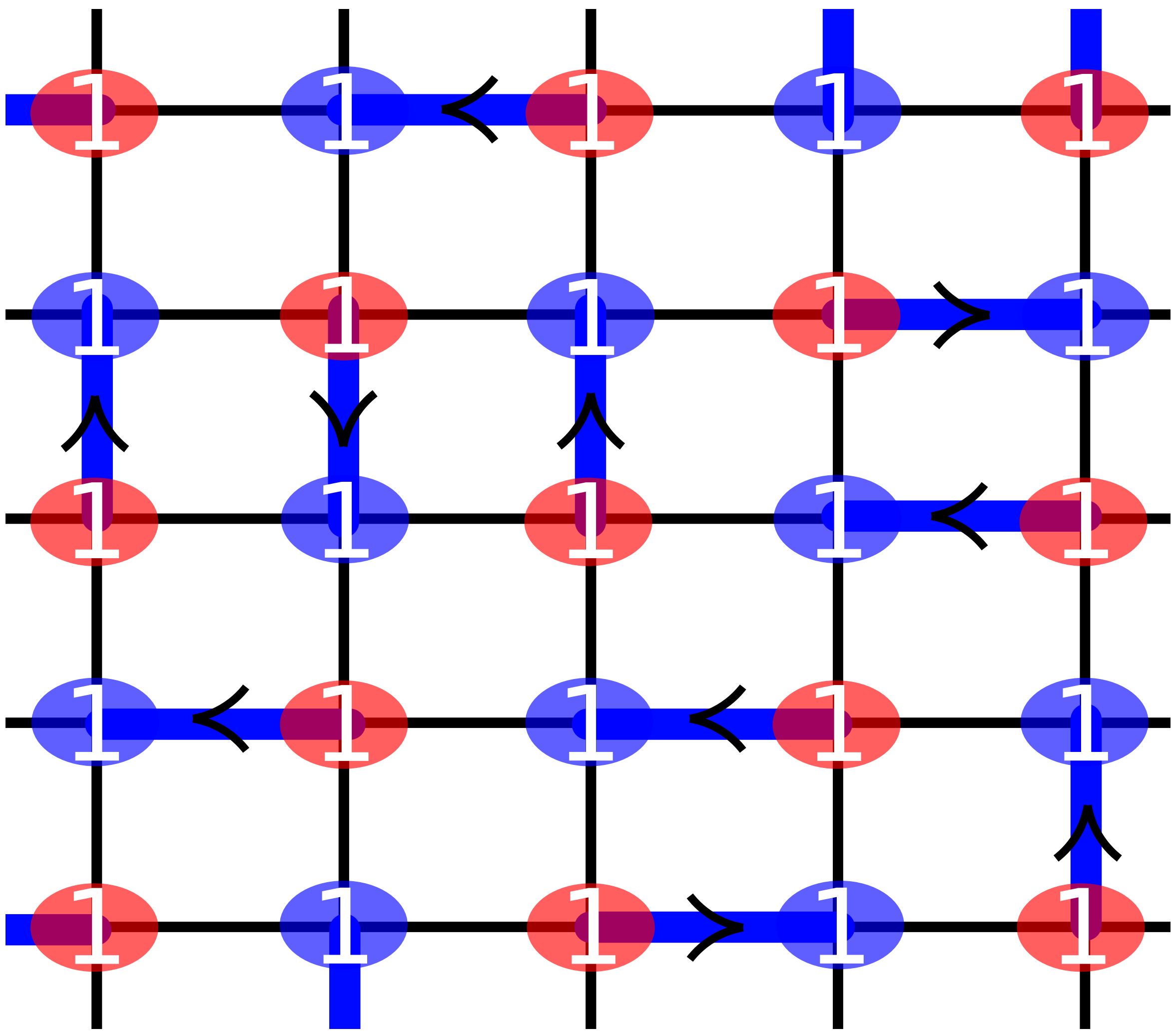

$$\xi \approx 2.76$$

Coupling $U(1)$ gauge field to charge $N$ matter field can produce $Z_N$ gauge theory.

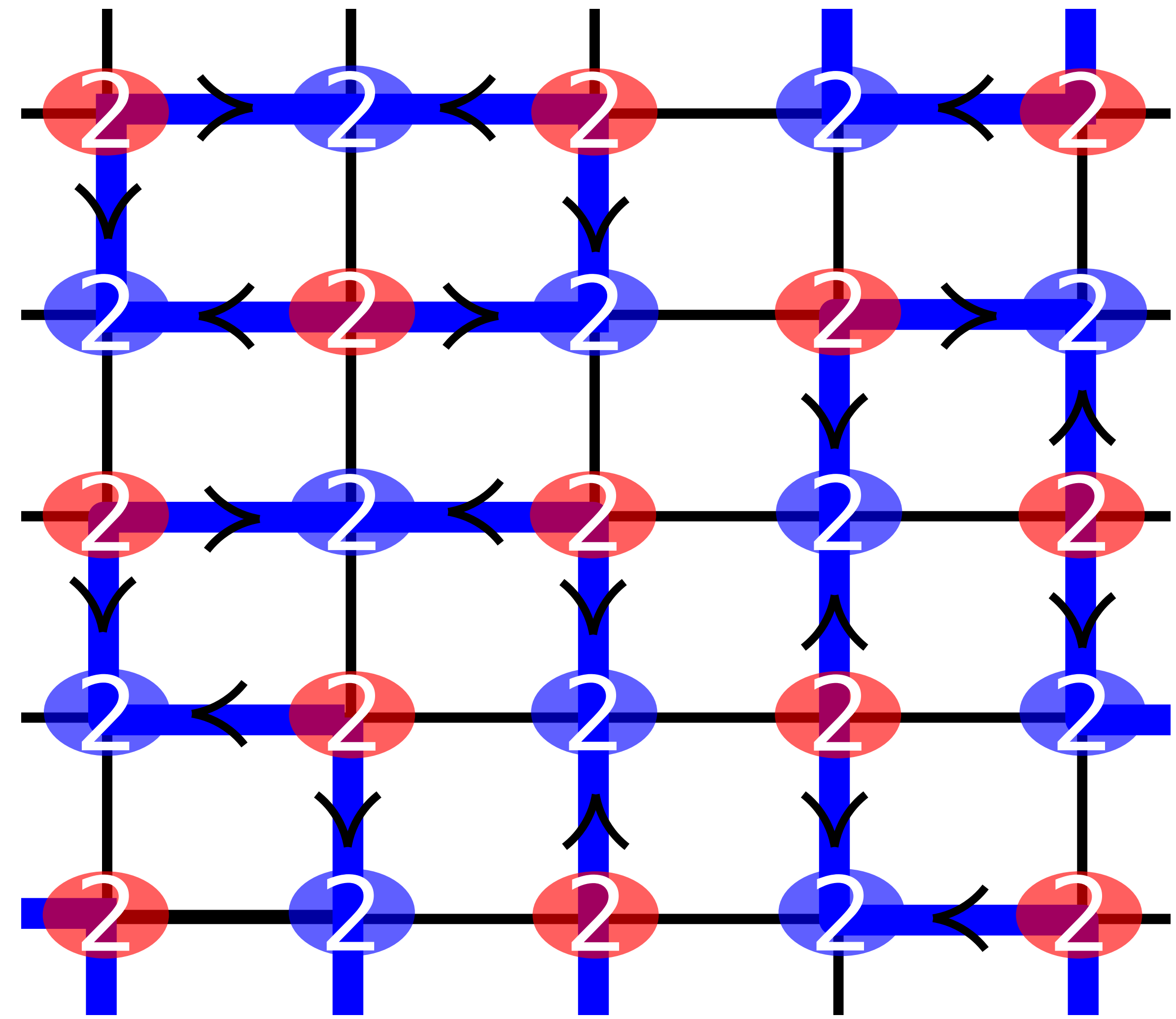
**Fradkin. Shenker. Phys. Rev. D. 1979.**

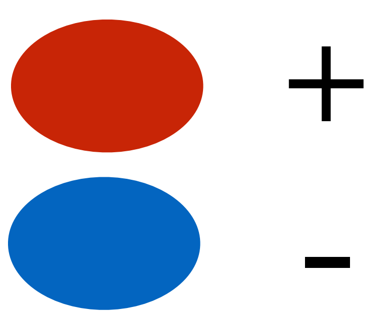
can produce zn gauge theory
(zN topological order)

## Quantum Dimer Model

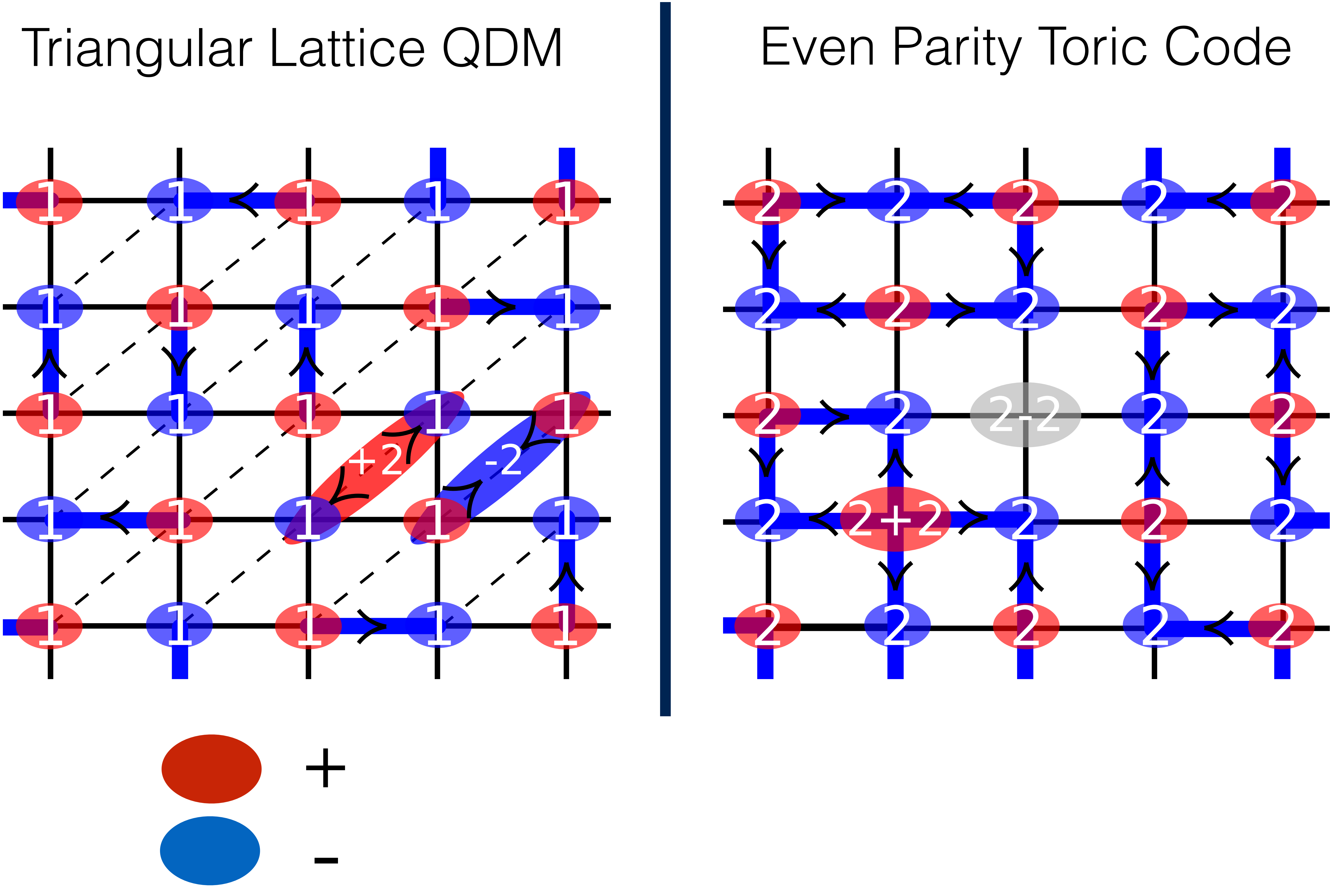


## Fully Packed Loop Model





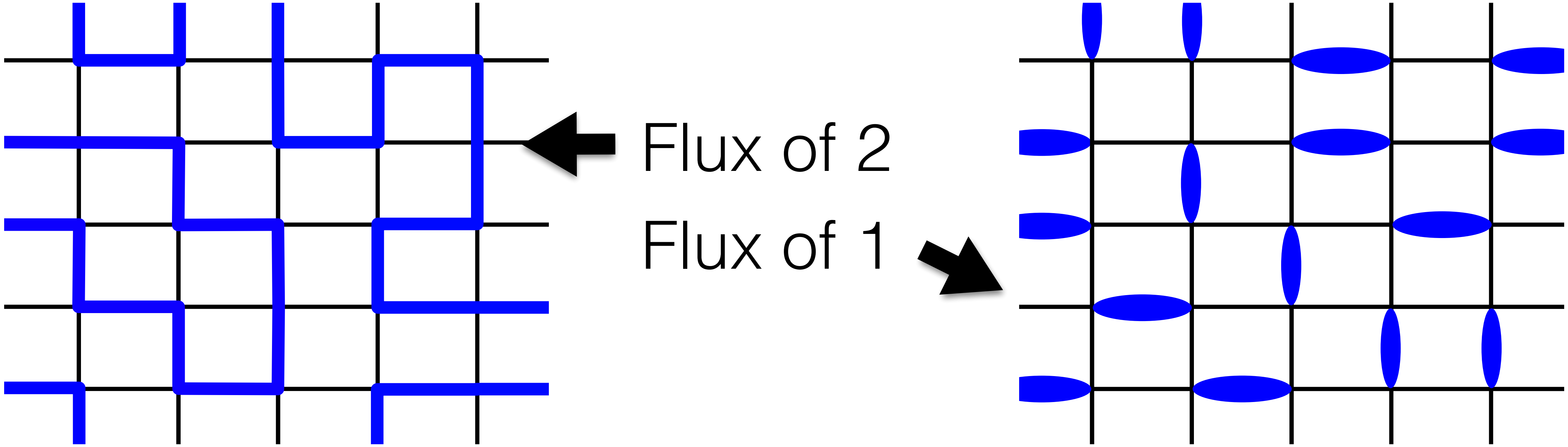

Dimer orientations are fixed by the lattice



Triangular Lattice QDM
Even Parity Toric Code
+2
-2
2-2
2+2
+
-

Fully packed loop model

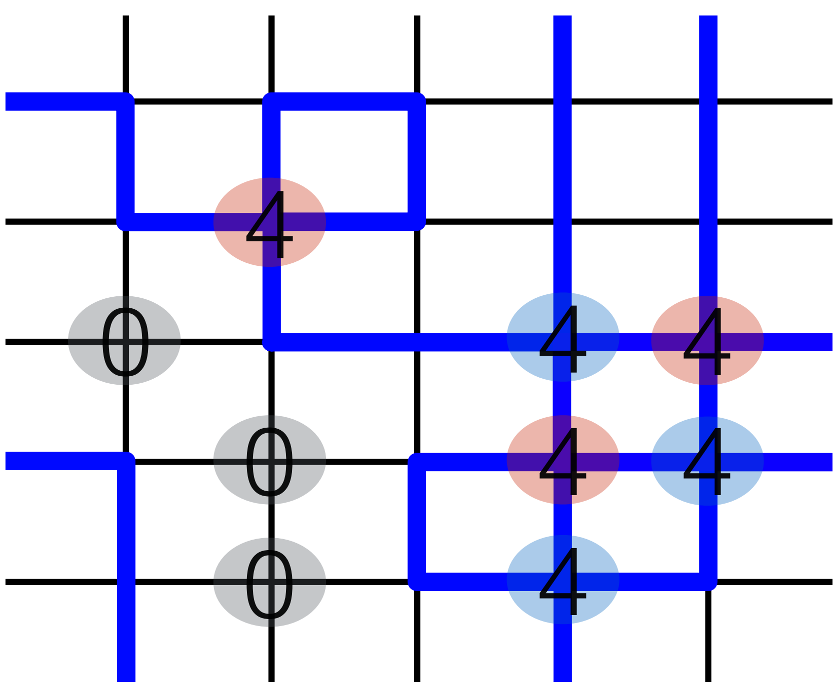
Square lattice QDM




$U(1)$ gauge field coupled to charge 2 $\rightarrow$ $Z_2$

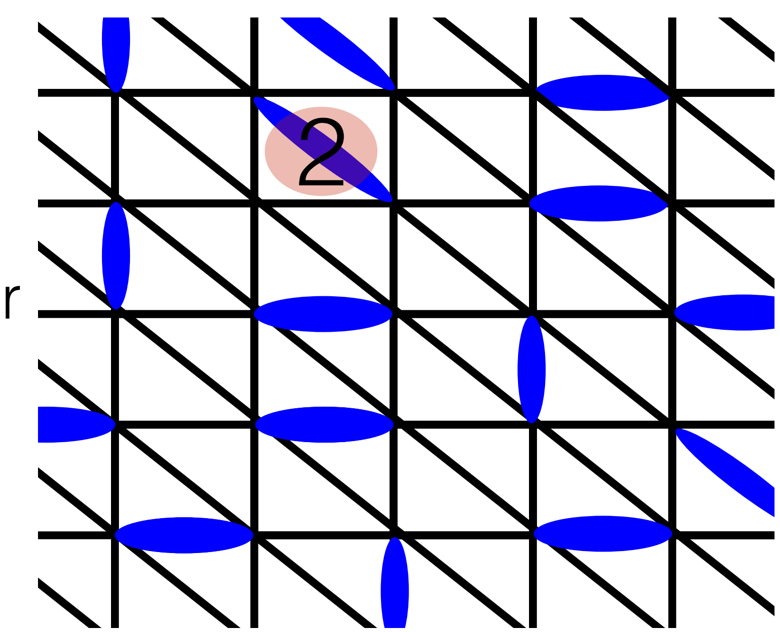
Toric code

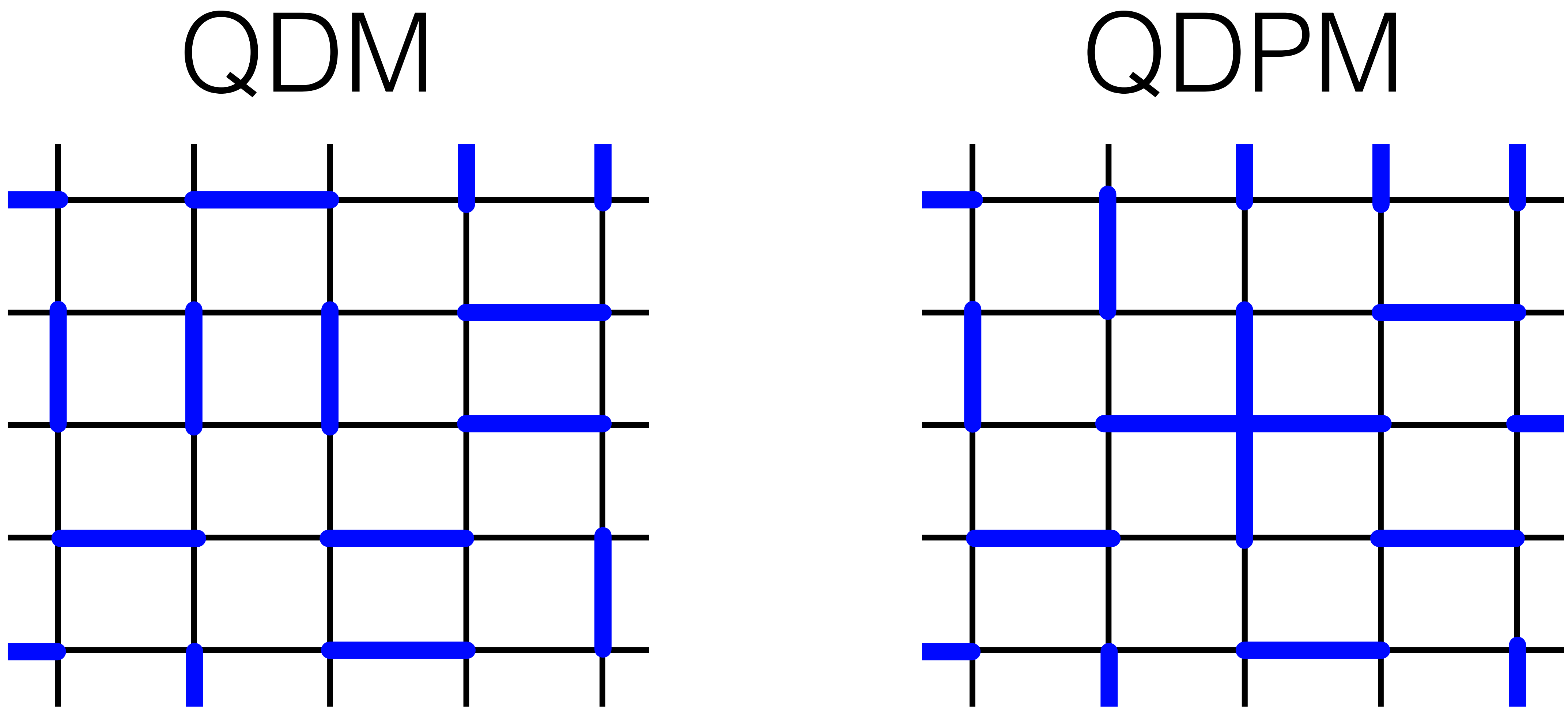
Triangular Lattice QDM



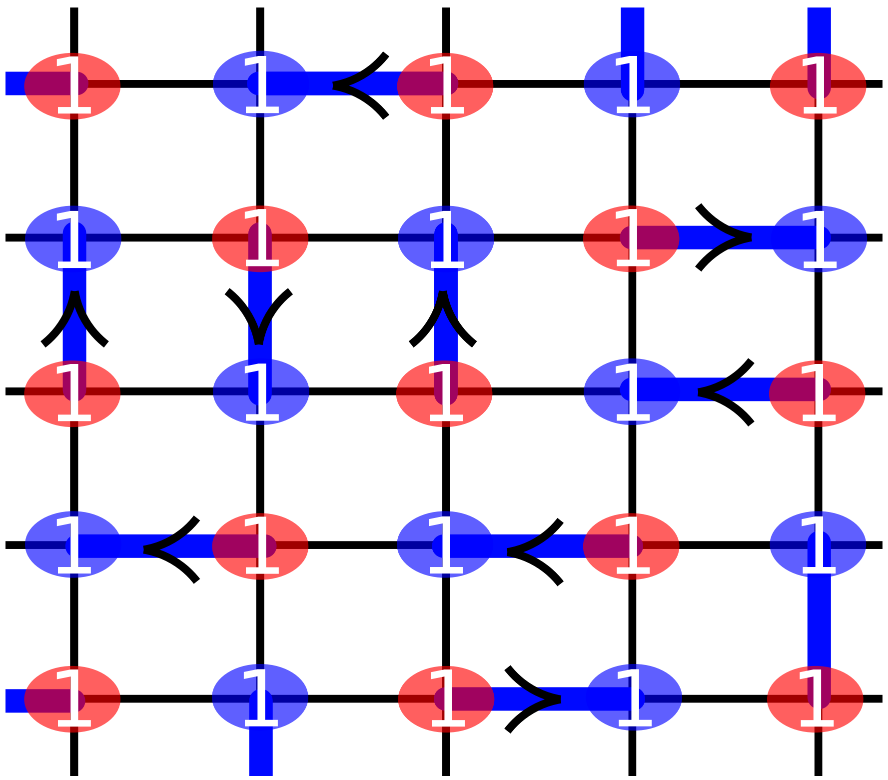

(At the RK Point)

- Gapped
- Exponential decay of dimer correlations in liquid state
- Finite topological degeneracy

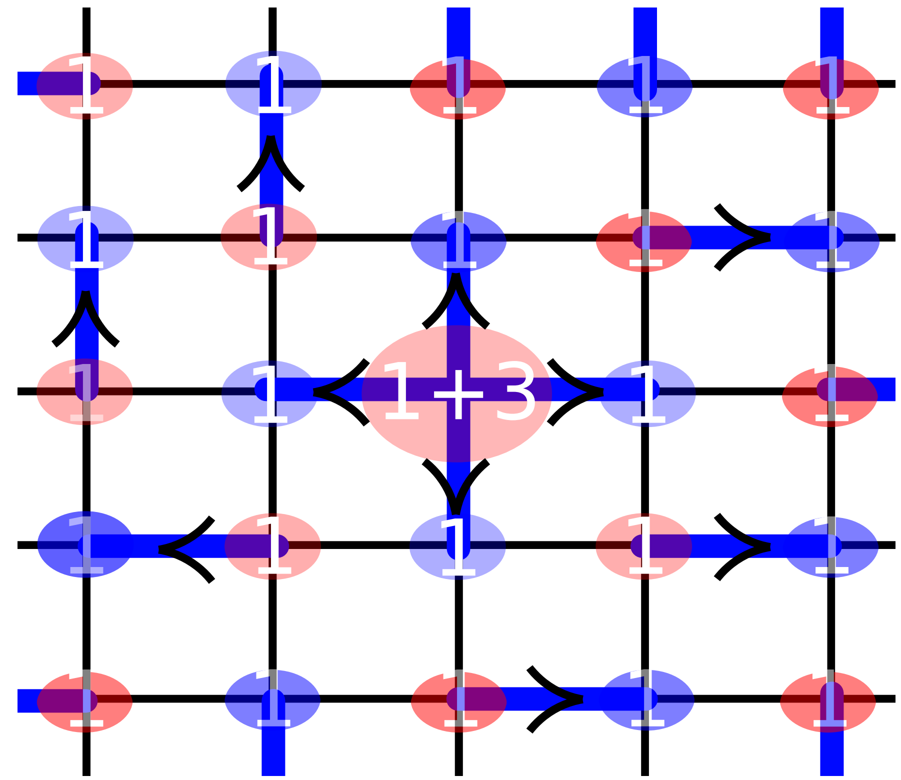



QDM
QDPM

# QDM

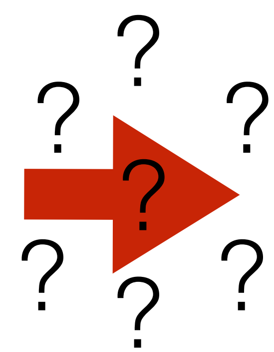


# QDPM




$n_v = 1, 4$ $\qquad e^{i\alpha(n_v - 1)}$ $\qquad \alpha = \{0, 2\pi/3, -2\pi/3\}$

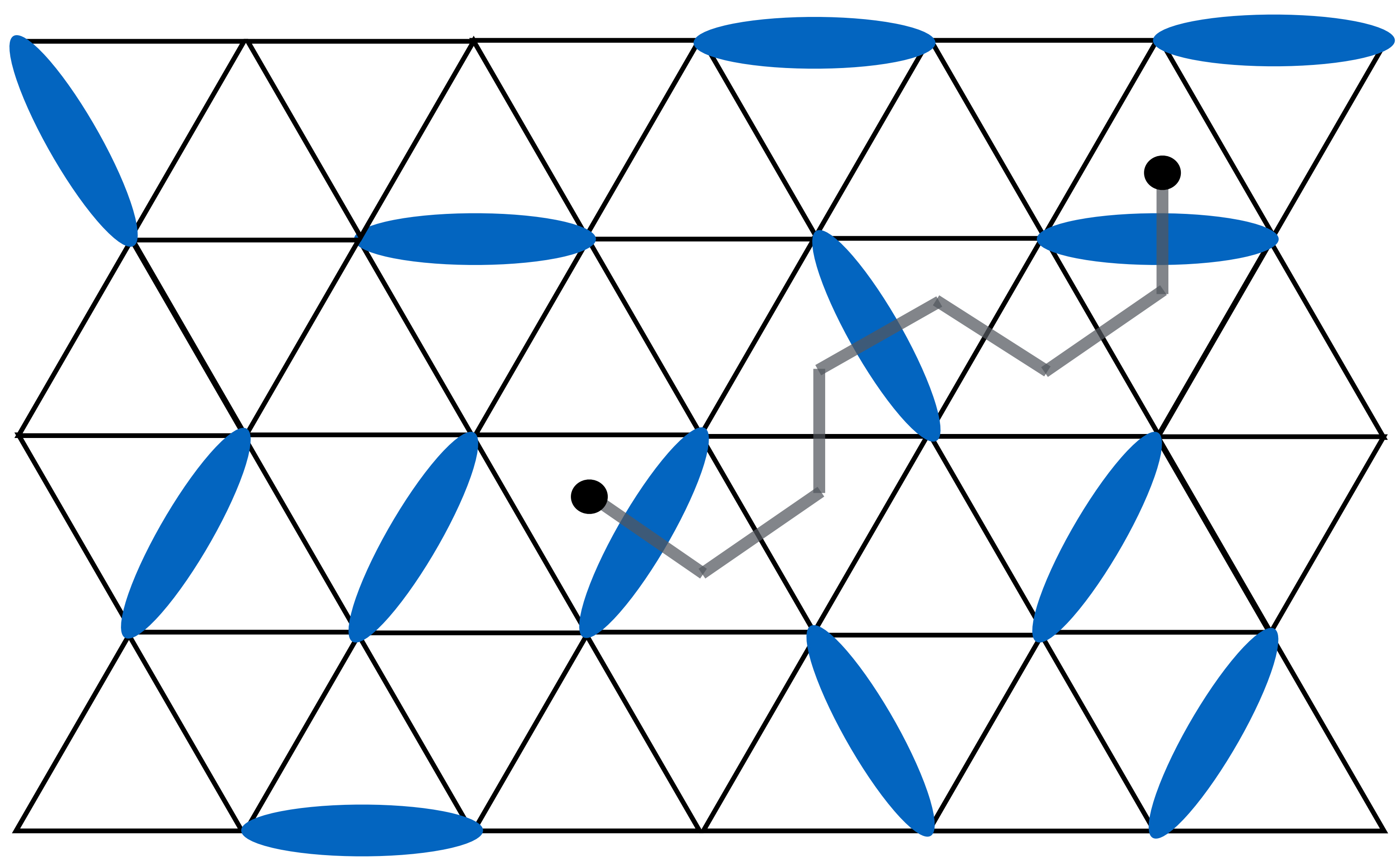
Local $Z_3$ gauge symmetry




$Z_3$ topological order?

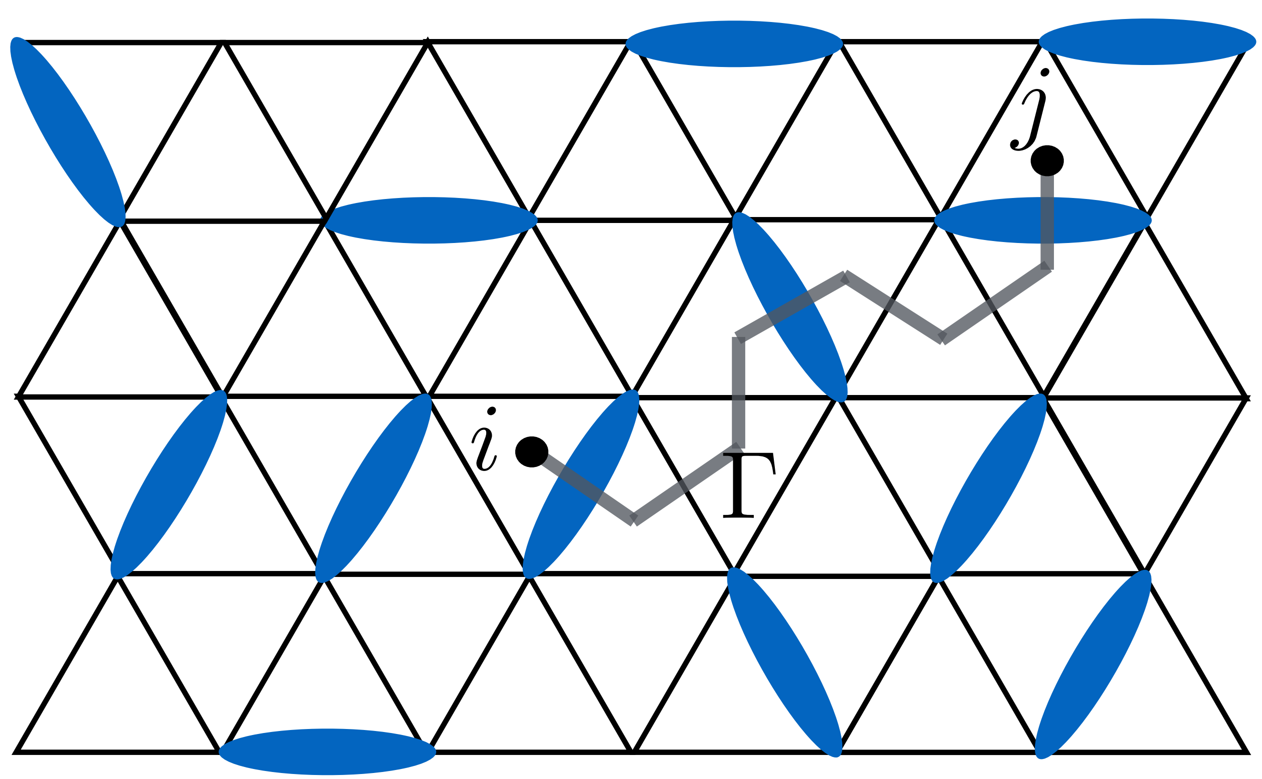
What is the nature of the lowest lying excited states in the QDPM?

# Visons in the triangular lattice dimer model



$$V_i V_j = \prod_{l \in \Gamma} e^{i\pi n_l}$$

- $\Gamma$ is the path. Open path makes 2 visons $\rightarrow$ $V_i V_j$
- lowest lying excitations
- Closed string commutes with the Hamiltonian
- $V_i V_j |\Psi_0\rangle$ $\rightarrow$ variational excited state
- $Z_2$ topological order




Vison imaginary time correlation function

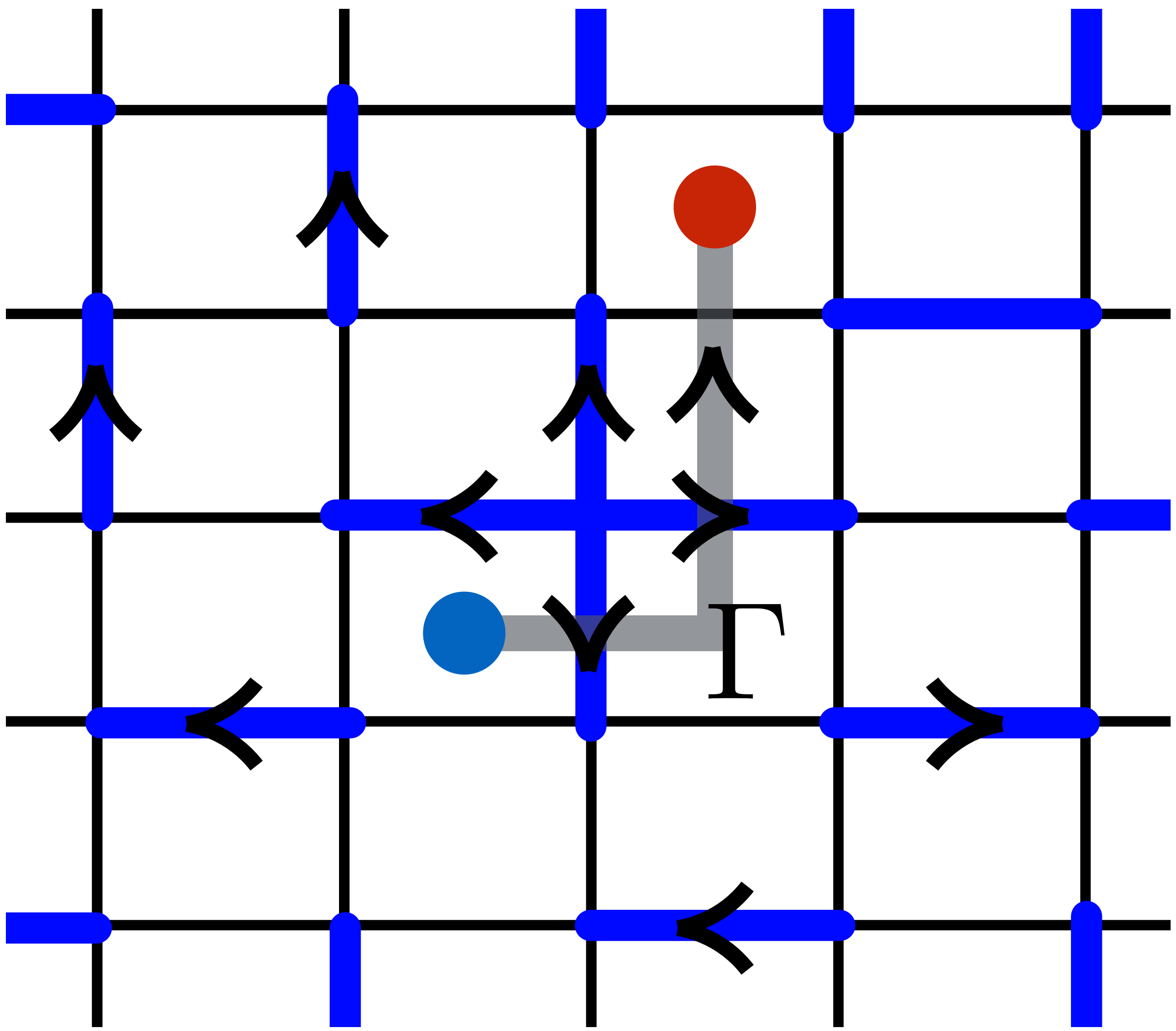
$$\langle V_i(0) V_j(\tau) \rangle$$

**Ivanov, Phys. Rev. B. 2004.**

# QDPM "$Z_3$ Visons"

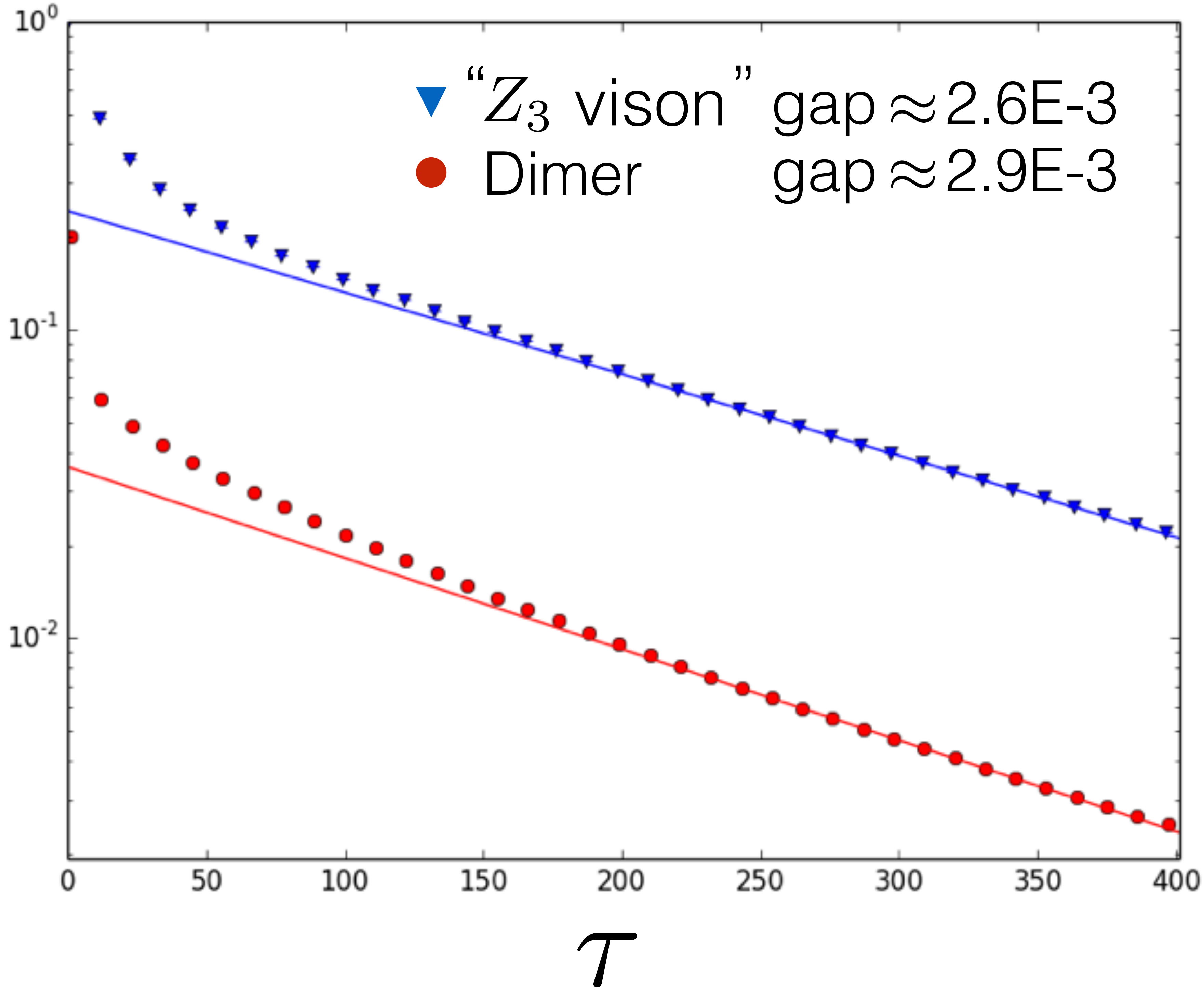
$$V_i V_j = \prod_{l \in \Gamma} e^{i\frac{2\pi}{3}(\pm n_l)}$$

Sign depends on the orientation of the link.




- Oriented string and two magnetic charges.
- Closed string commutes with the Hamiltonian
- Open string makes variational excited state

# Imaginary time correlations




Extract gap from imaginary time correlations.

# Conclusions

- We propose a quantum dimer-pentamer model which may exhibit $Z_3$ topological order.
- We show evidence of a dimer liquid state at the RK point.
- Using a Monte Carlo method we sample the ground state wave function at the RK point to calculate the dimer and “$Z_3$ vison”correlations.
- We show that the vision gap and dimer gap are of similar size.

# Future Work

- Resolve the vision and dimer gaps.
- Investigate signatures of topological order in the entanglement entropy.
- Determine the phase diagram away from the RK point.