Multiple particles' nonlinear dynamics in a spatiotemporaly periodic potential

- Advisors: Junru Wu, Jeffrey Marshall

- Funded by NASA Space Grant Consortium, NNX10AK67H and NNX08AZ07A

Outline

1) The electric curtain: Experimental, Numerical.

2) What are STP potentials?

3) Current methods of understanding.

4) The simple model: Work on the fundamentals.

5) Proposal.

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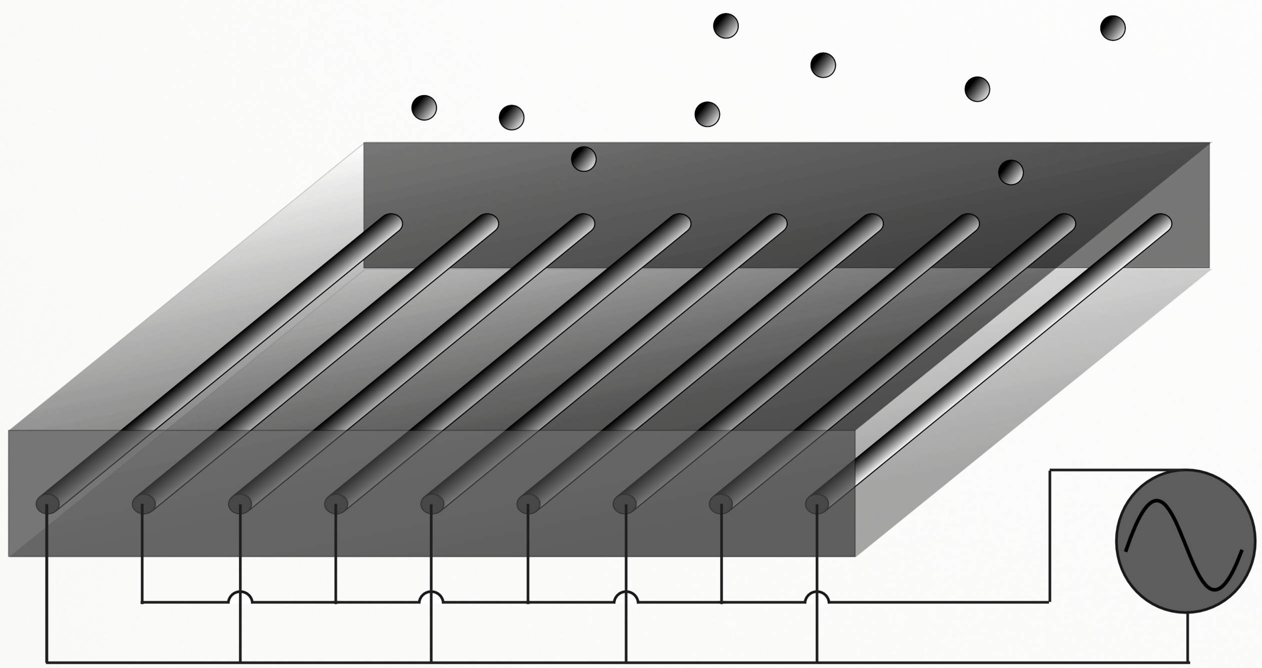
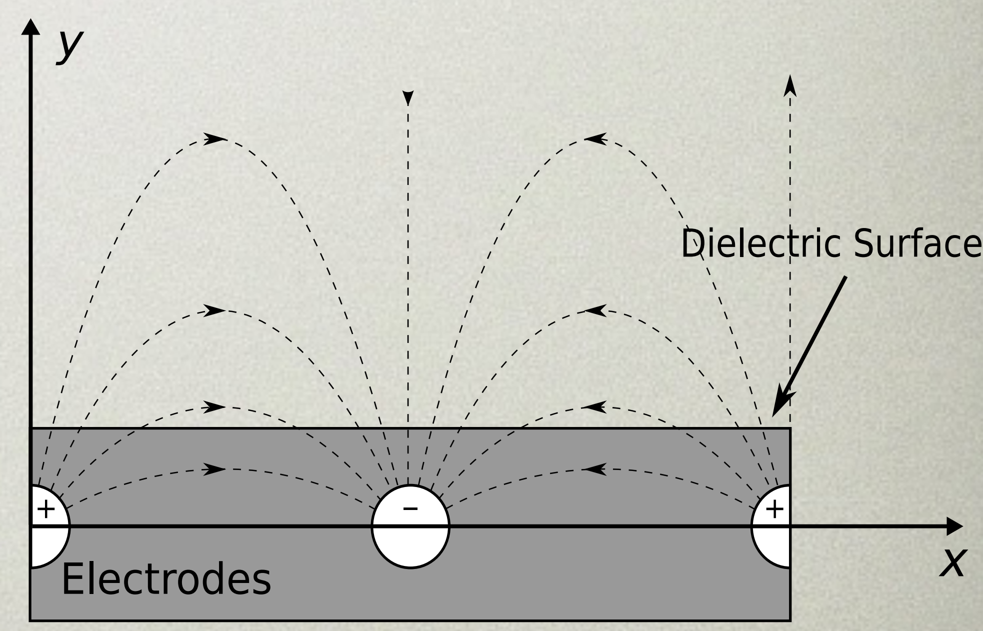
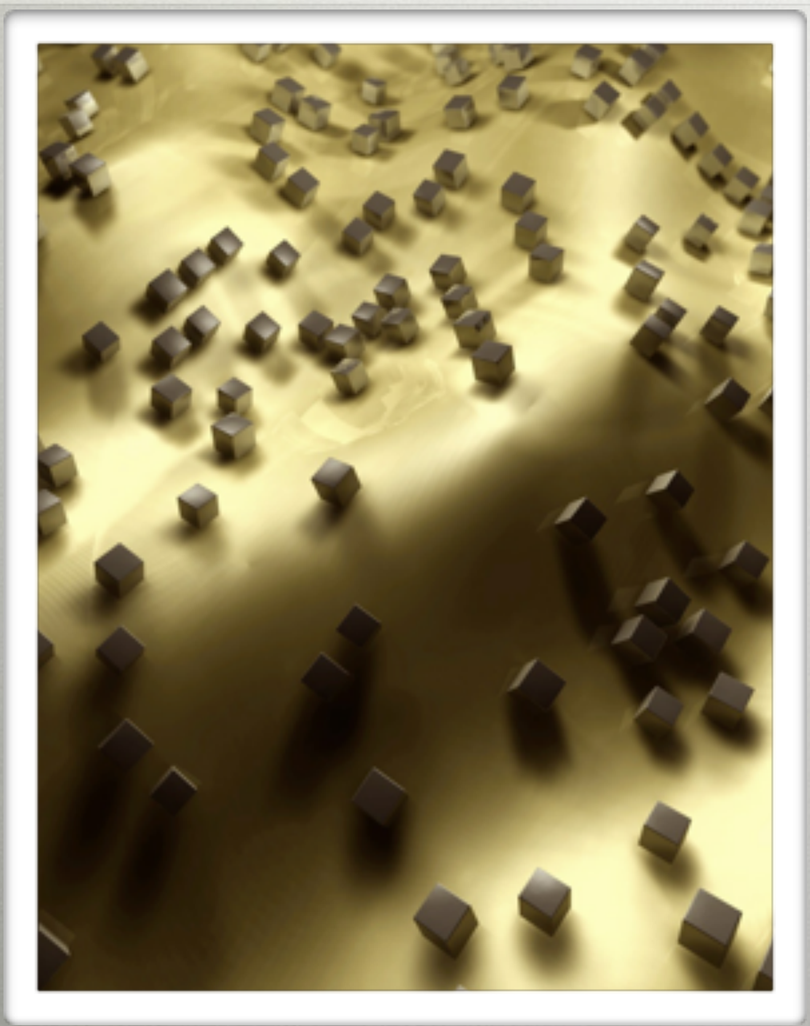
The Electric Curtain

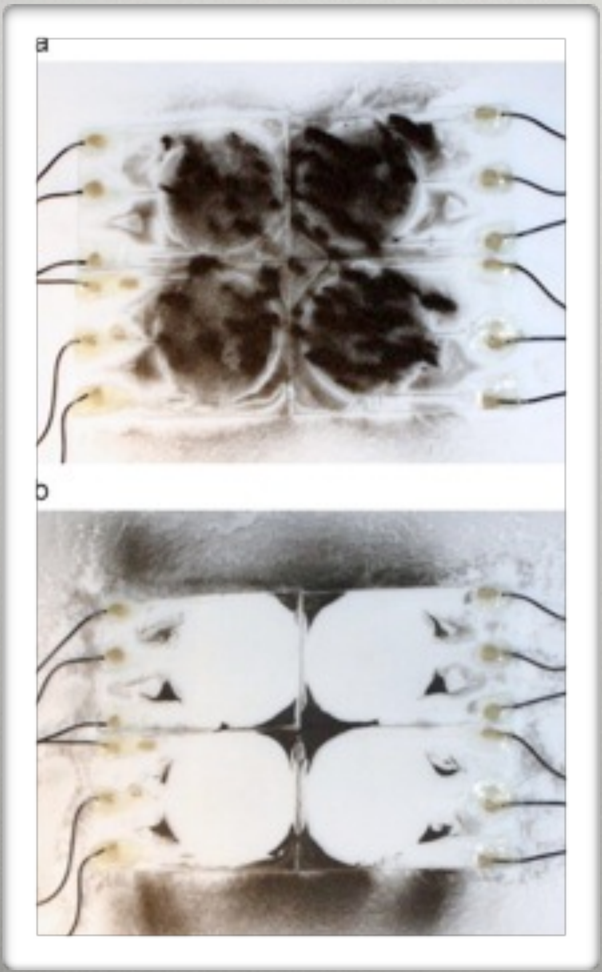


Experimental





A. Antoine, (Nature,2012).
Image: Cristian Ciraci



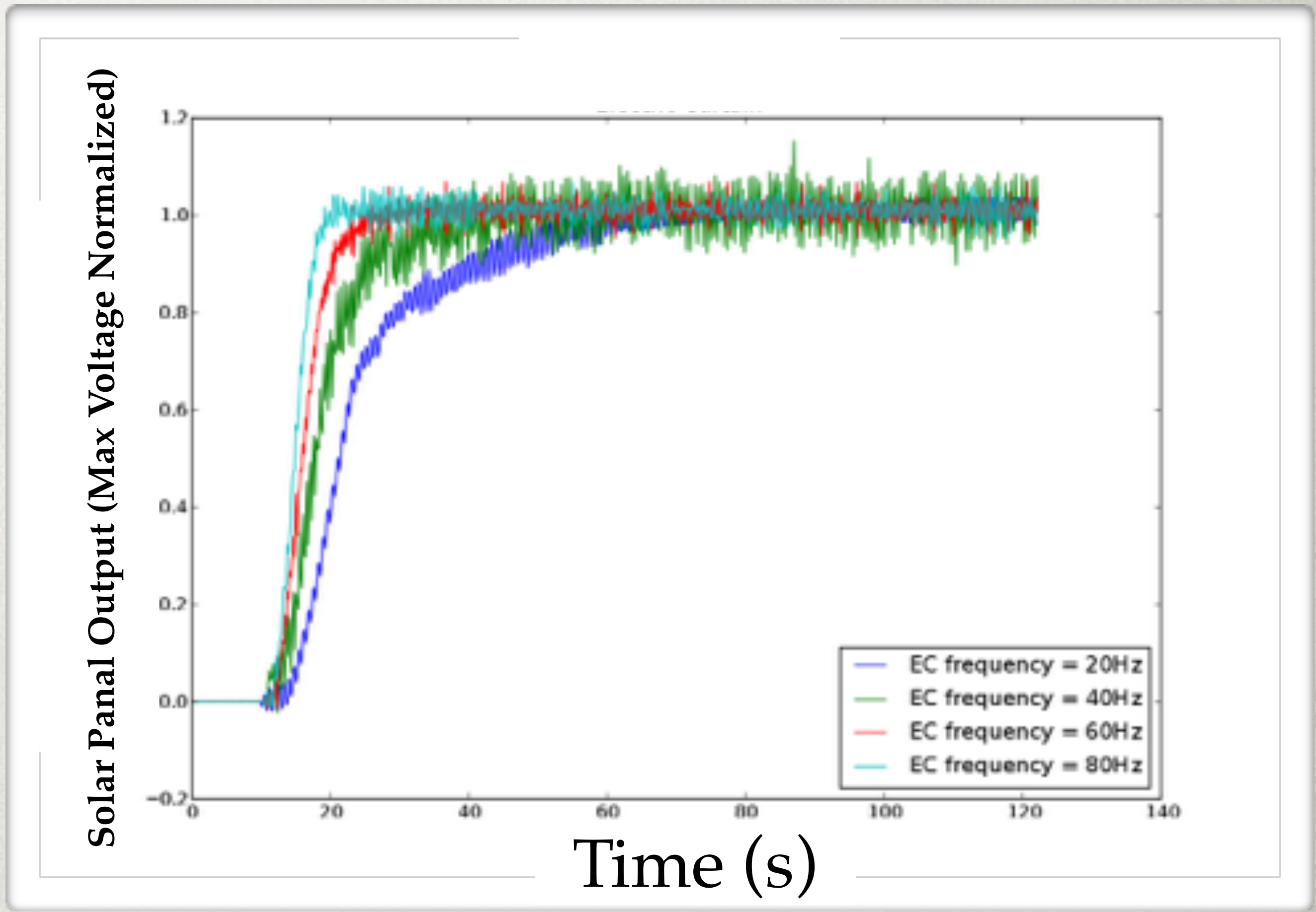
C.I. Calle
(Acta Astronautiva, 2011)

Masuda, Washizu and Kawabata (IEEE Trans. Ind. App. 1998)

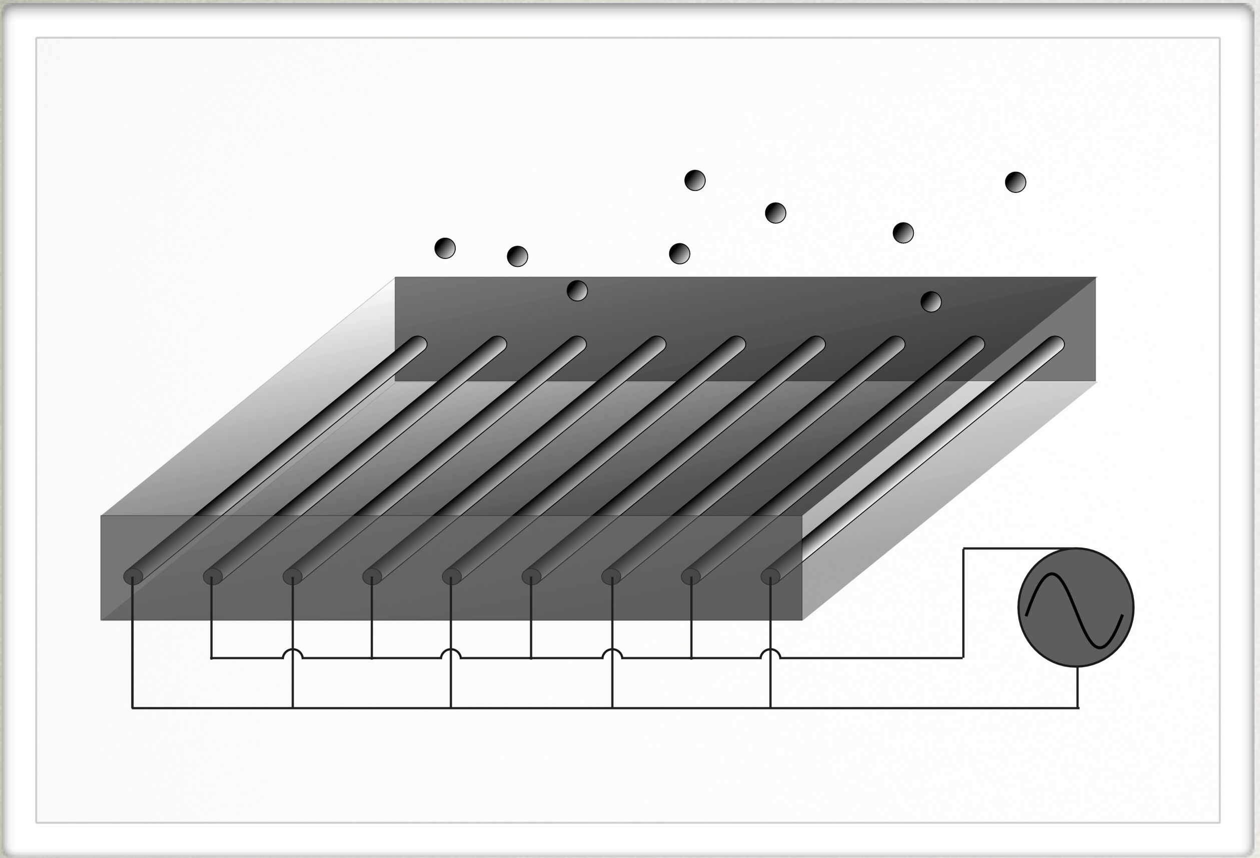
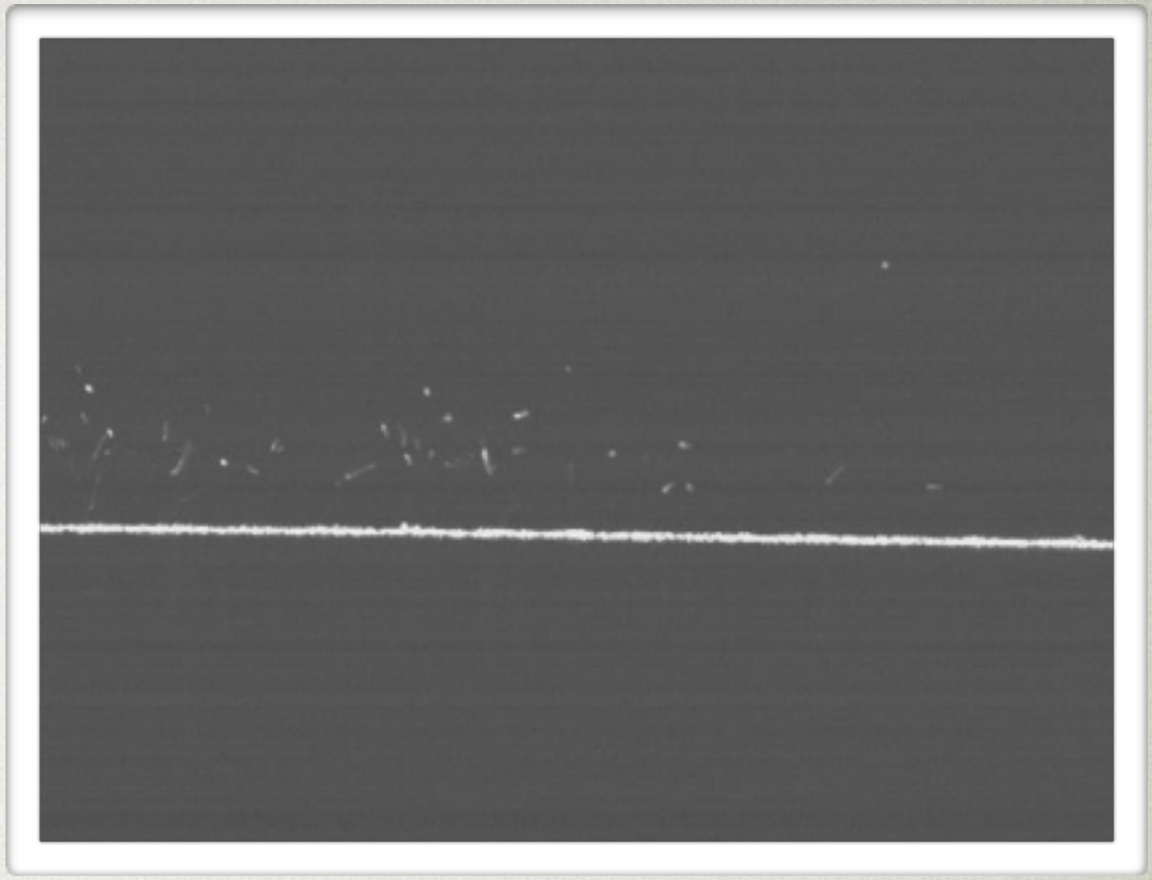
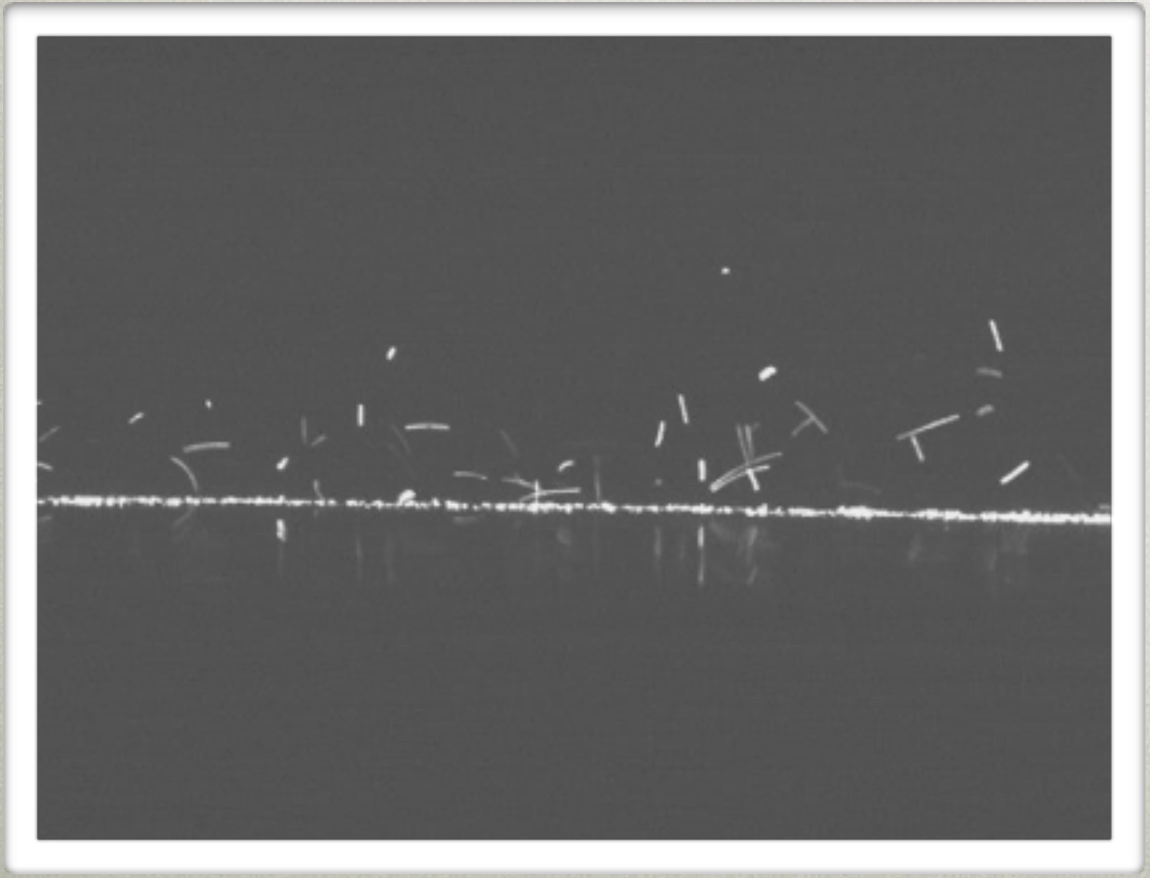
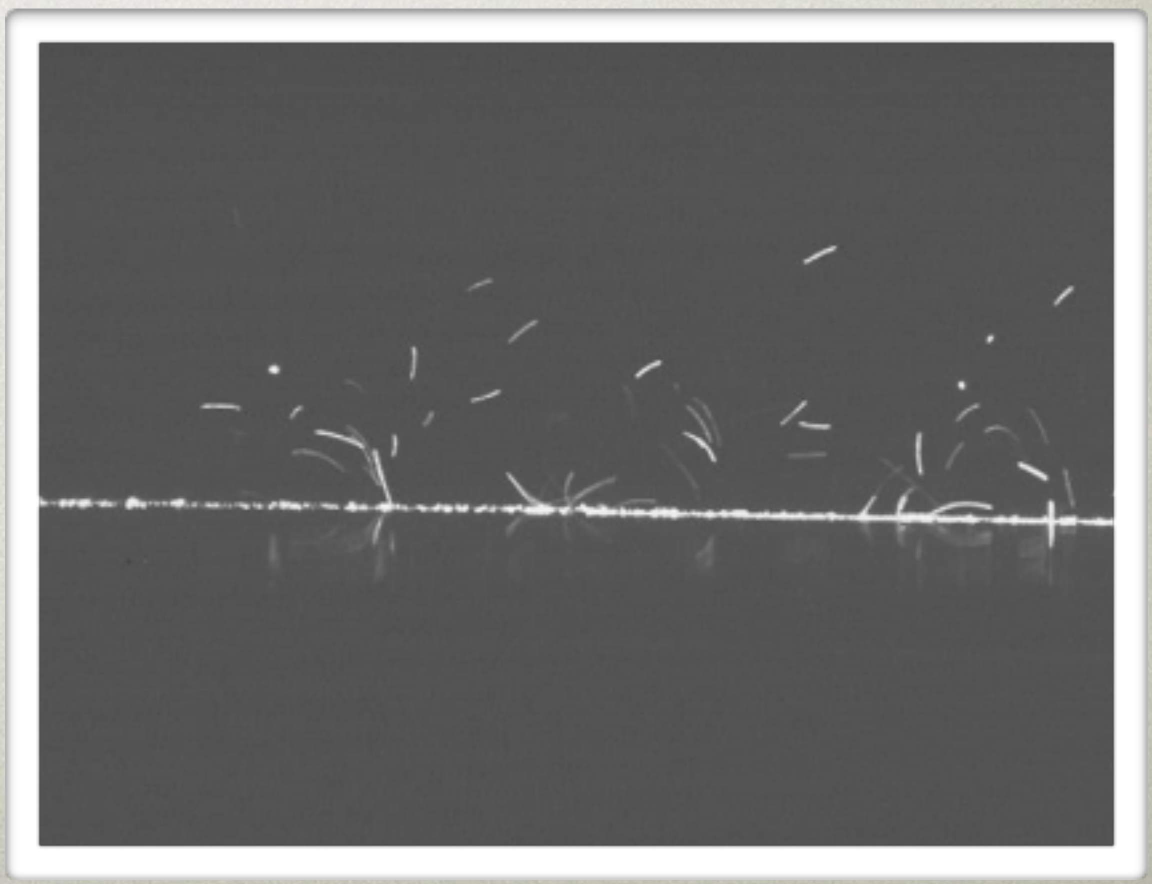
- Separation of particles
  H. Kawamoto, (2008)

- Liquid drop transport
  H. Kawamoto and S. Hayashi, (DATE)

Cleaning solar panels



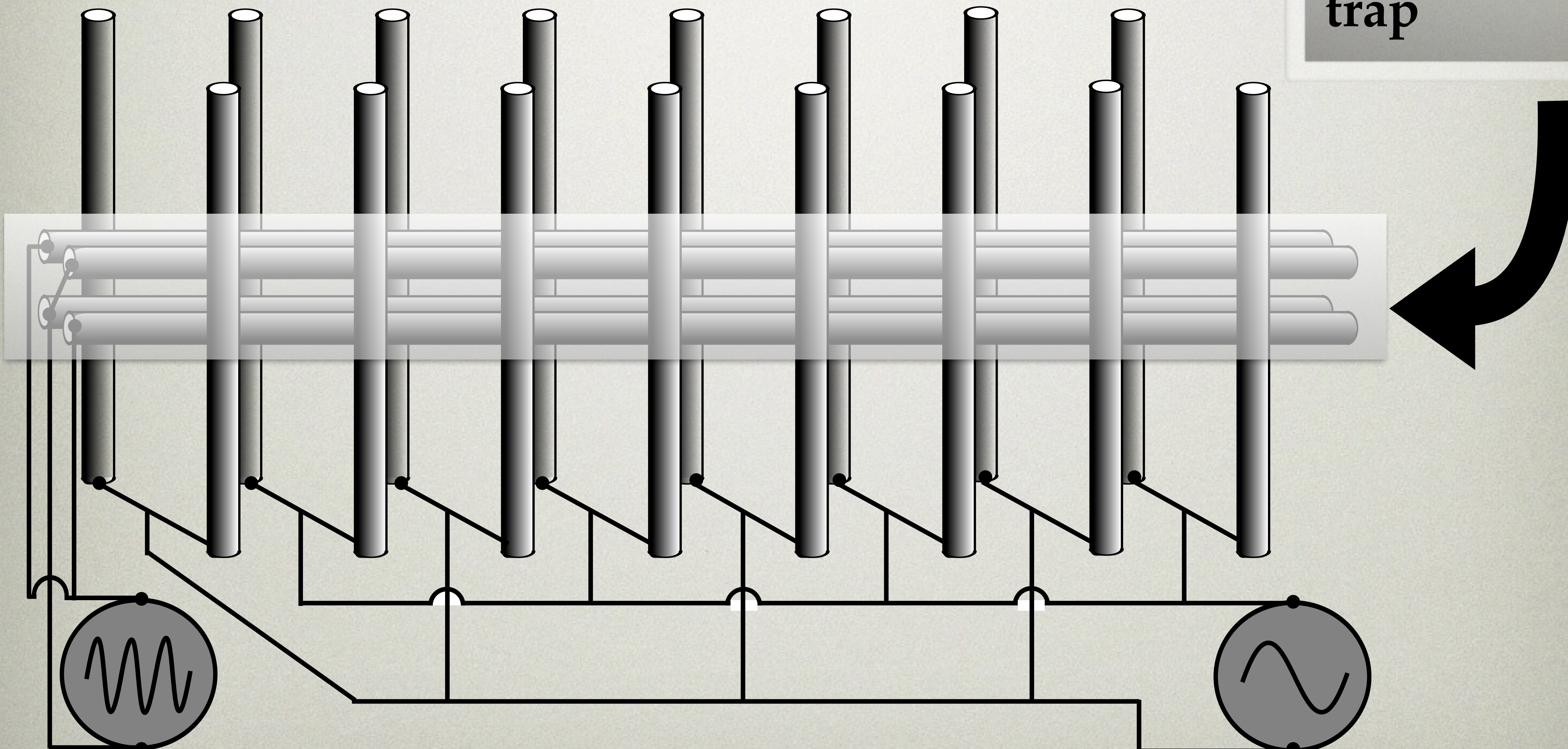
Complicated Problem of
many charged particles interacting

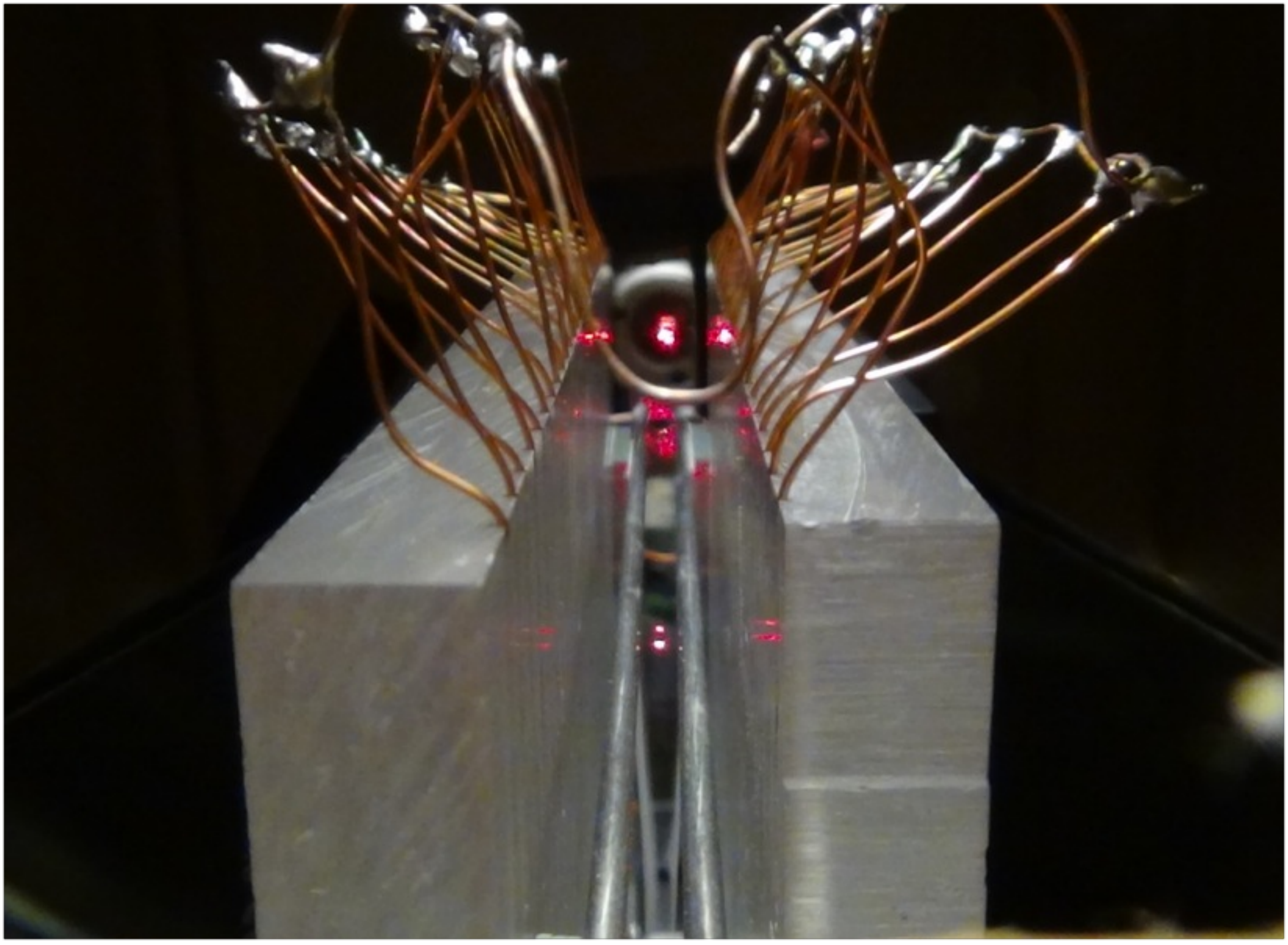
1D

Either constrain motion to surface or:

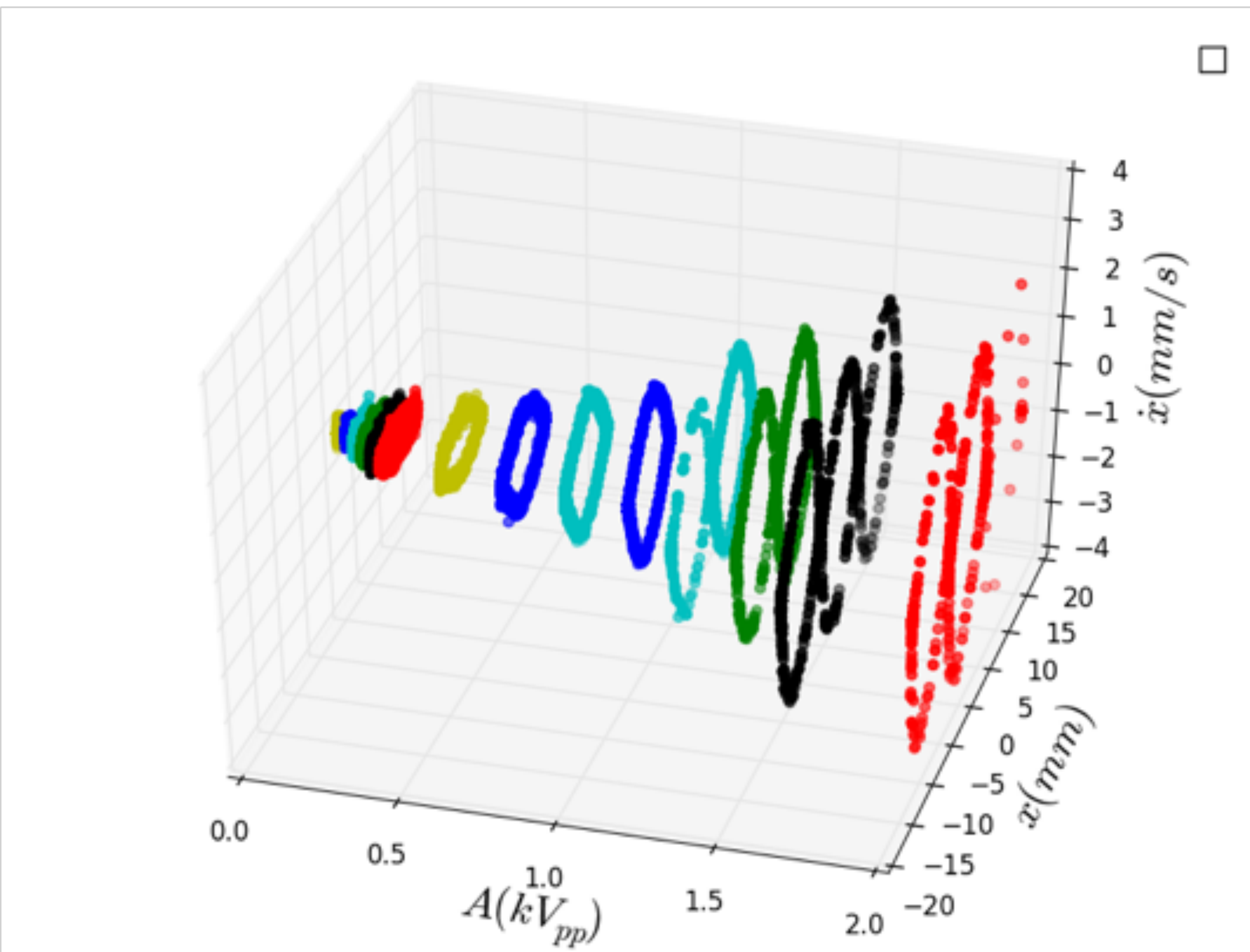
linear quadrupole trap



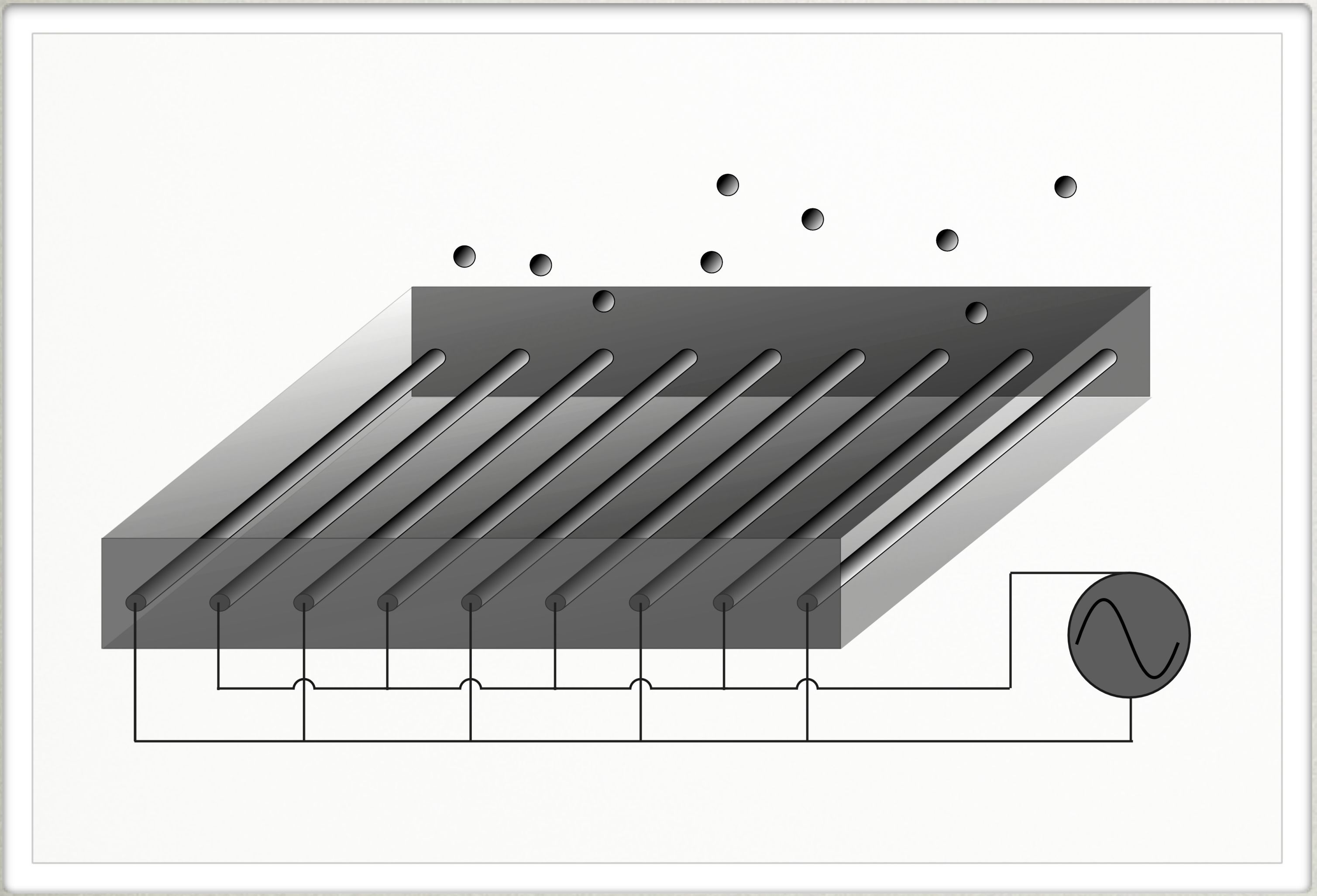
The actual set up



Single Particle Bifurcations



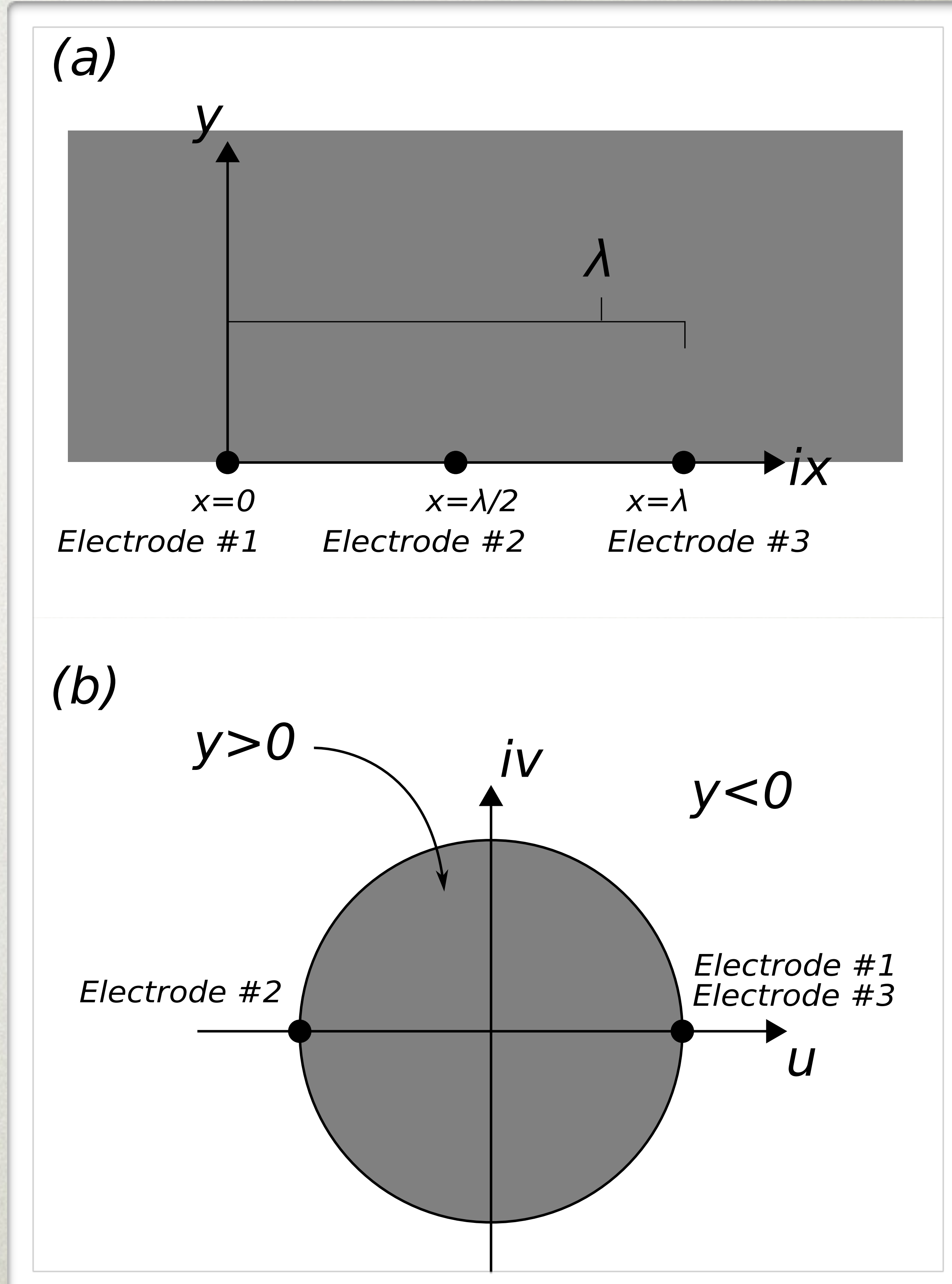
The Electric Curtain

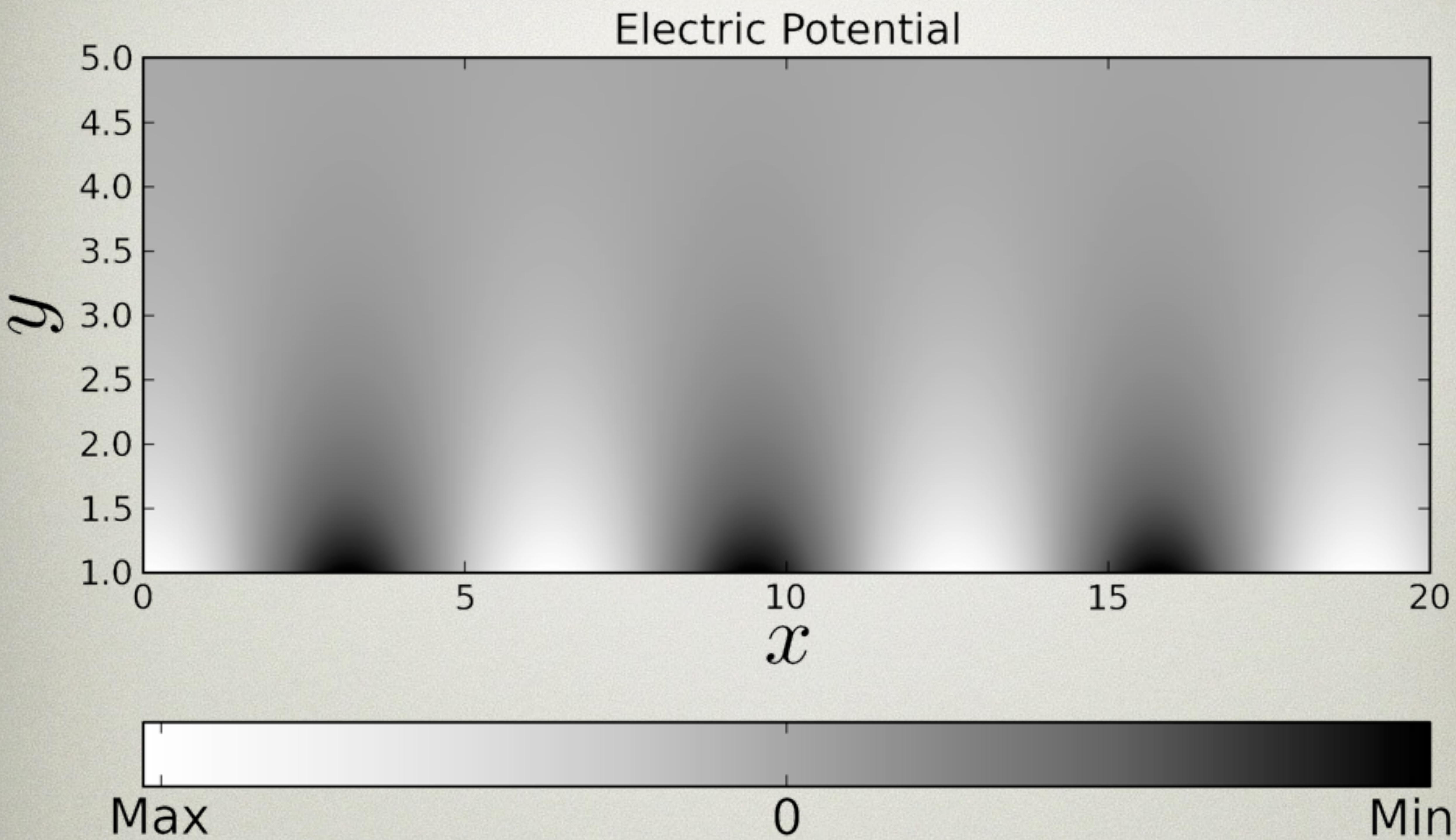


Experimental

Numerical

$$e^{(y+ix)2\pi/\lambda} = u + iv$$



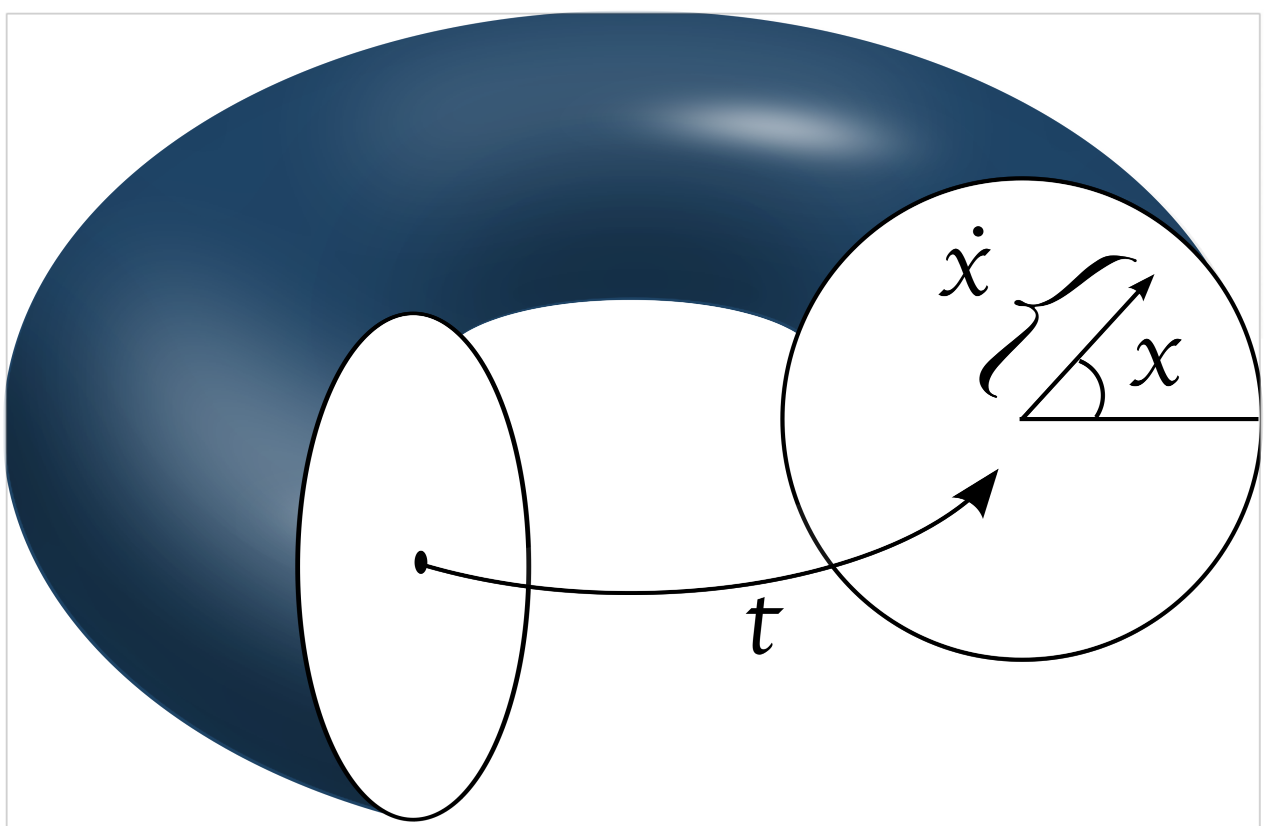


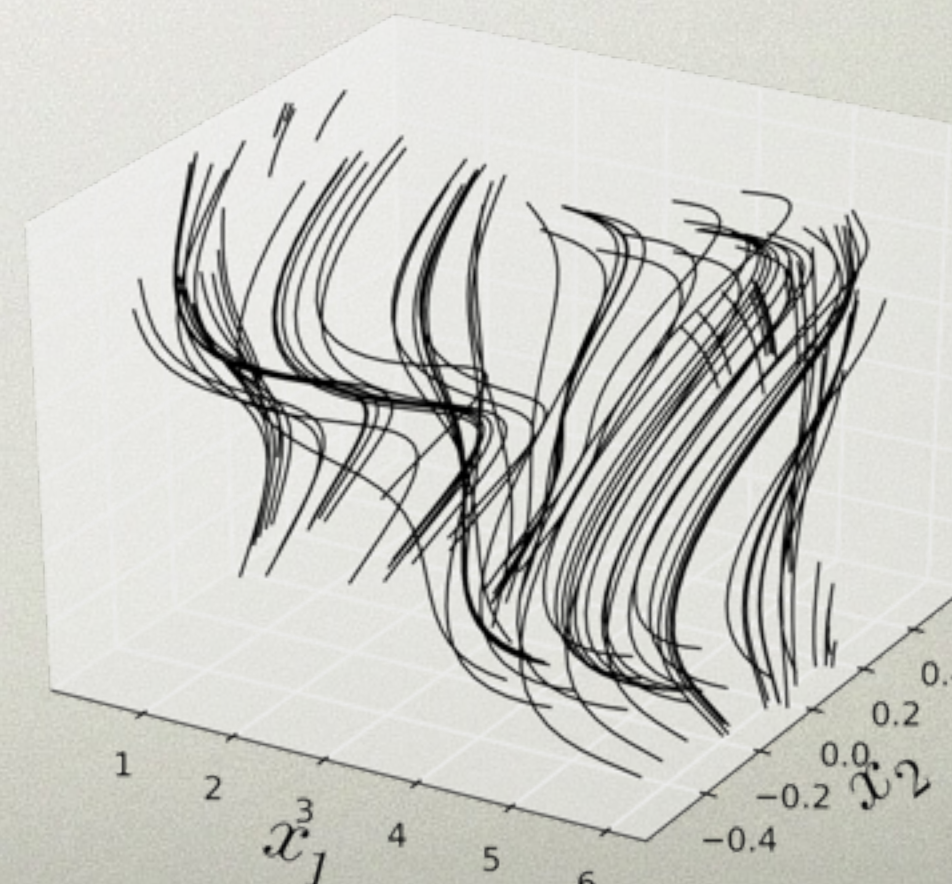
Electric Potential
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
y
0
5
10
15
20
x
Max
0
Min

1D

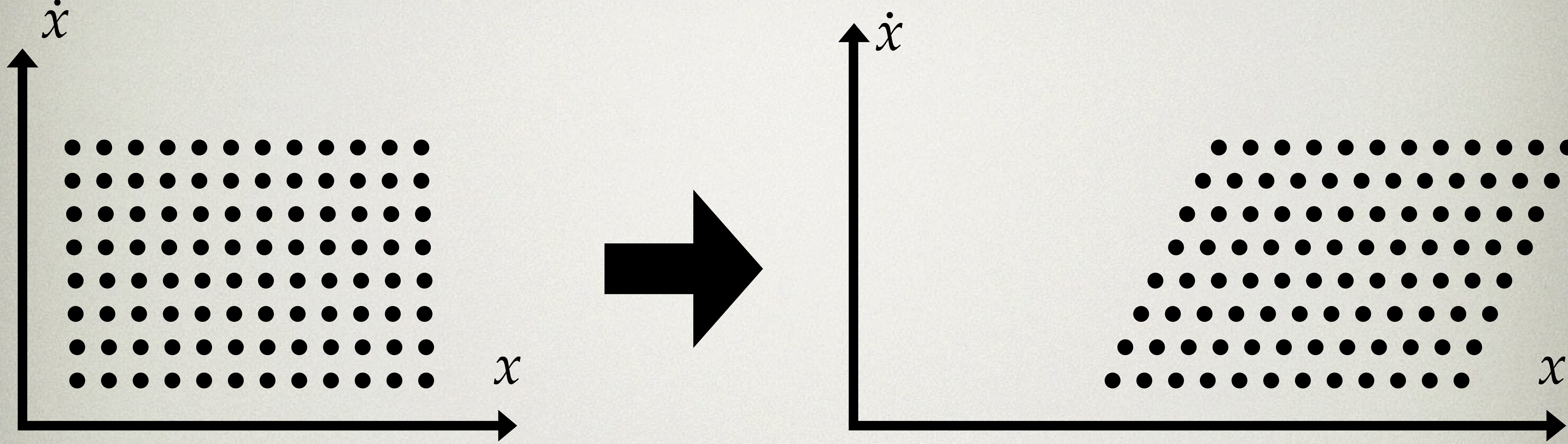
The Toroidal Phase Space

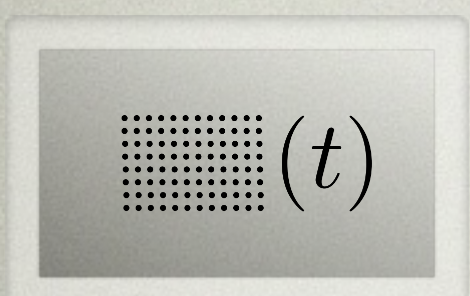
$$\dot{\mathbf{v}} = \mathbf{f}(\mathbf{v}) \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = A \sin x_1 \cos x_3 \left(\frac{2\cosh y}{\cosh^2 y - \cos^2 x_1}\right) - \beta x_2 \\ \dot{x}_3 = 1 \end{cases}$$

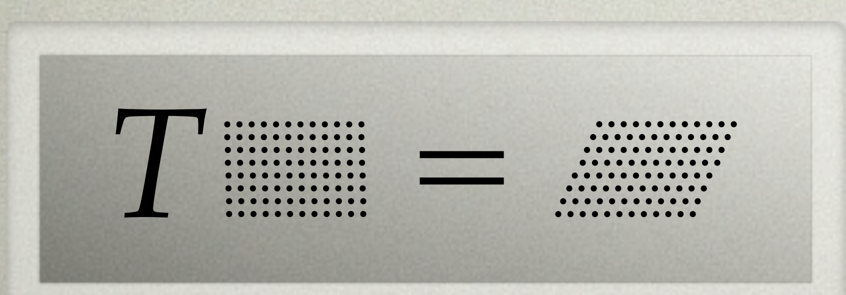




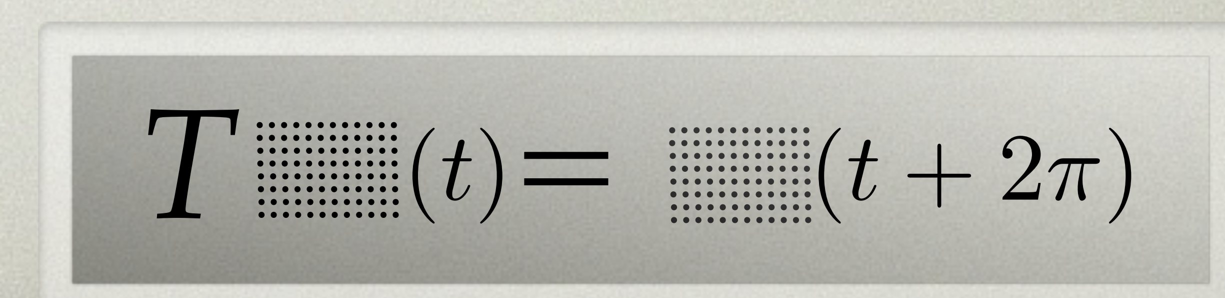
Time mapping and Poincaré sections







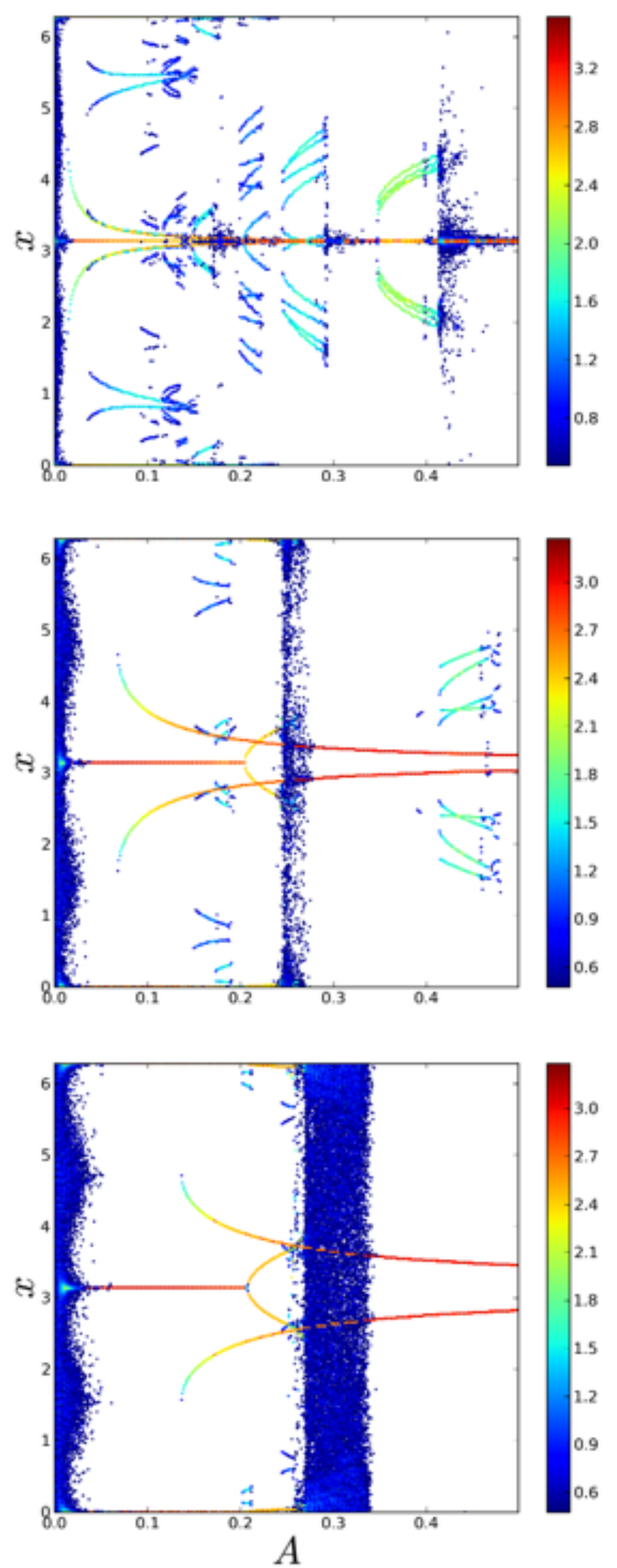
Poincaré Section

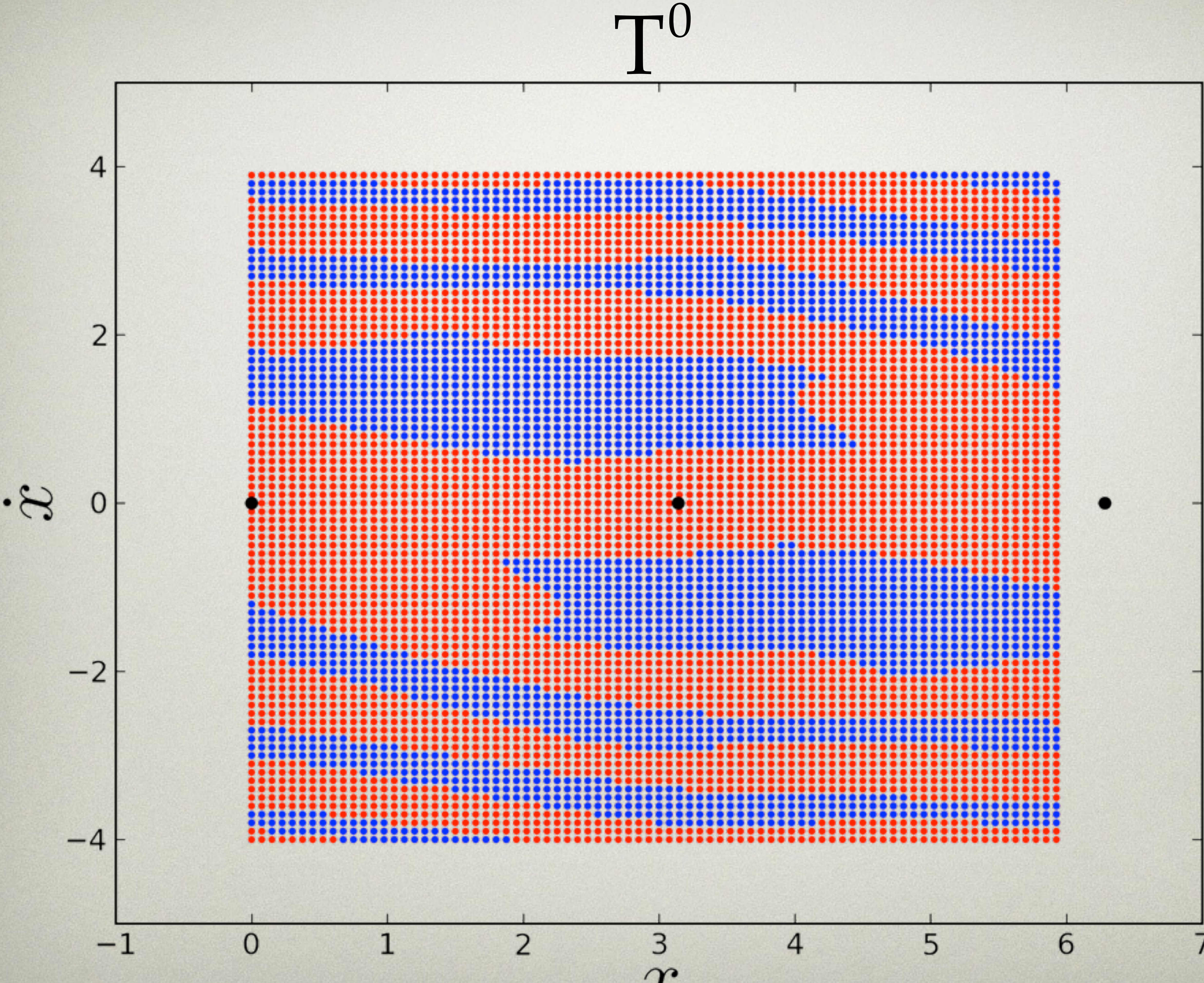


$\beta = 0.01$

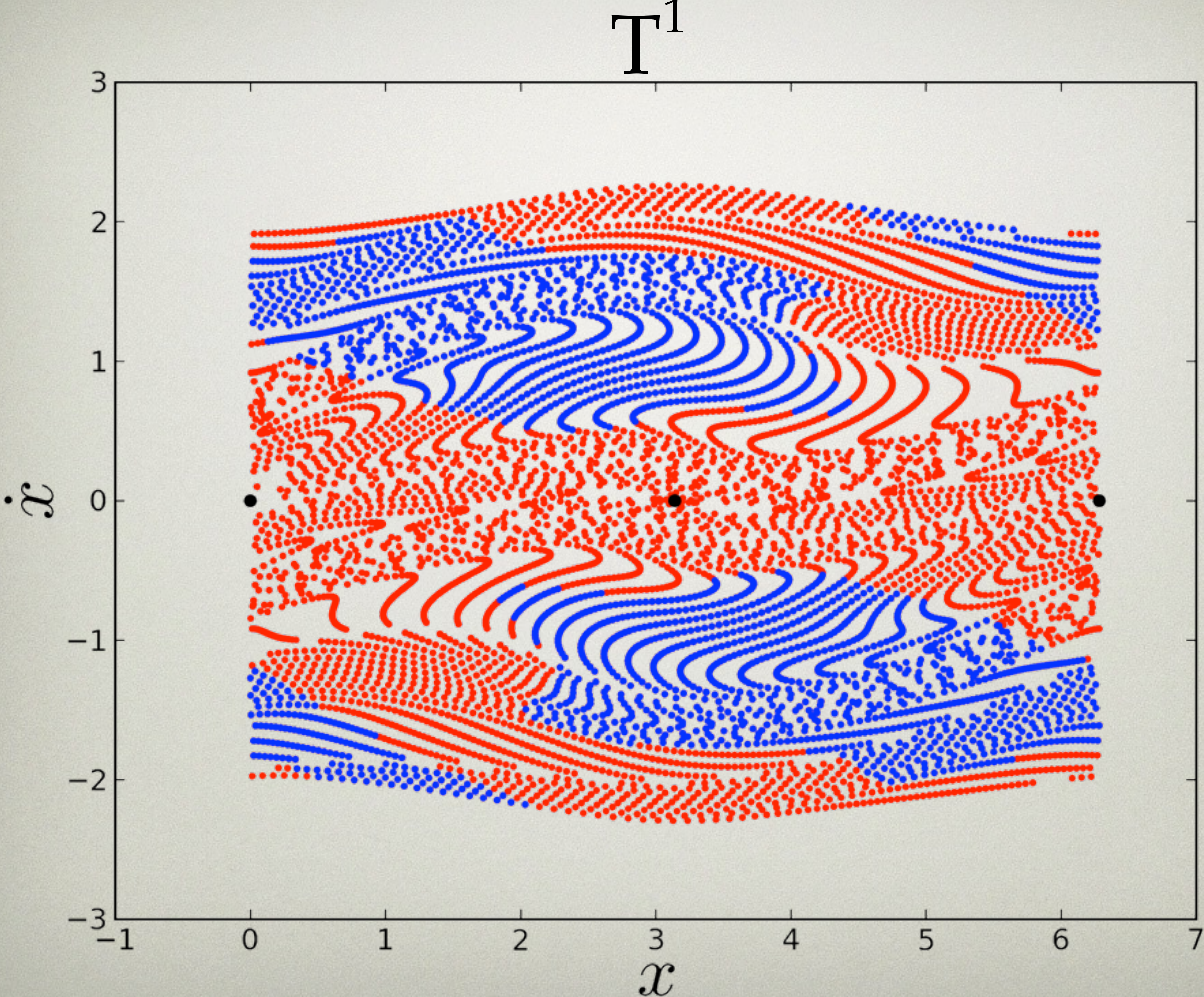
$\beta = 0.05$

$\beta = 0.1$

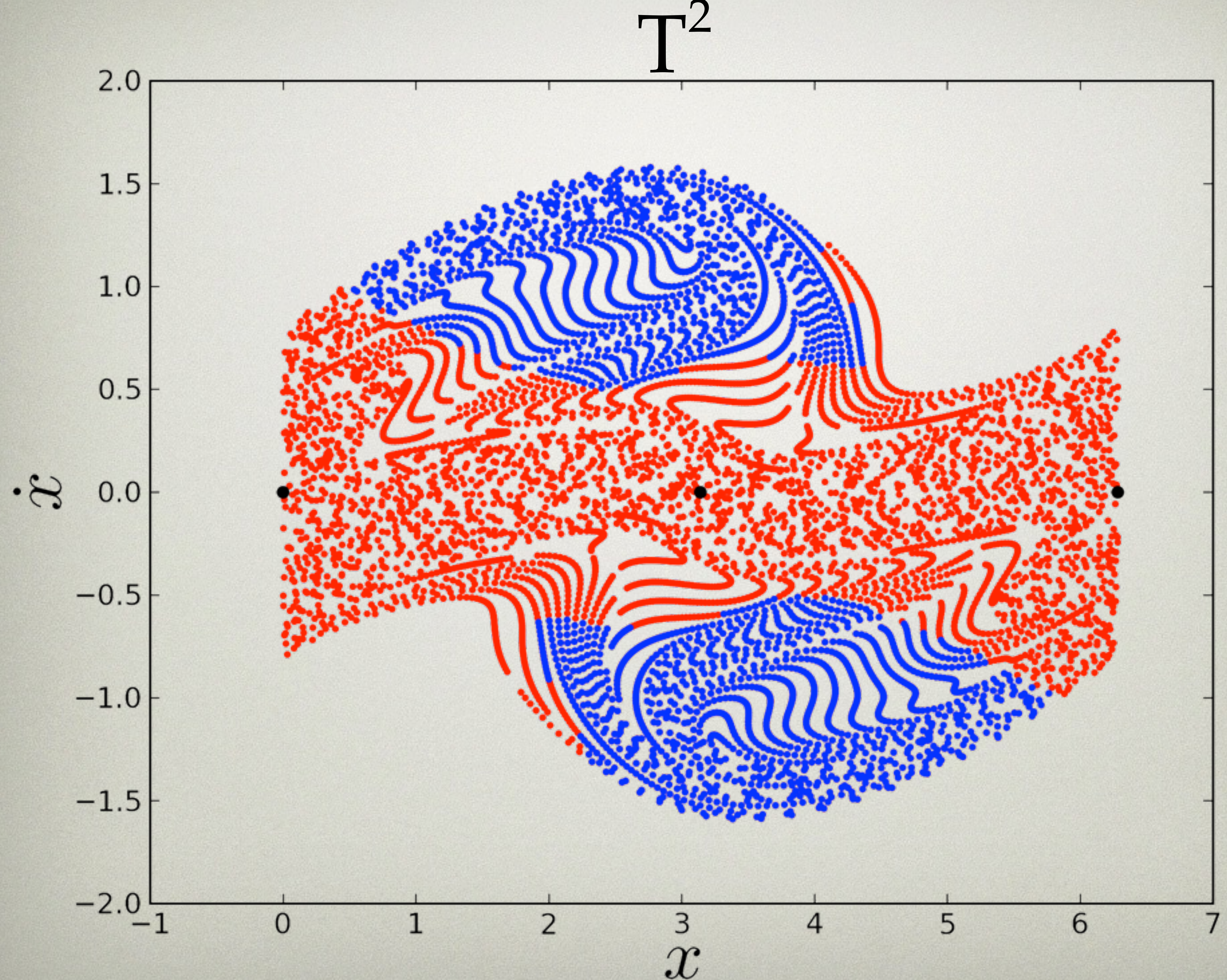




T⁰
4
2
0
−2
−4
−1
0
1
2
3
4
5
6
7
x
ẋ

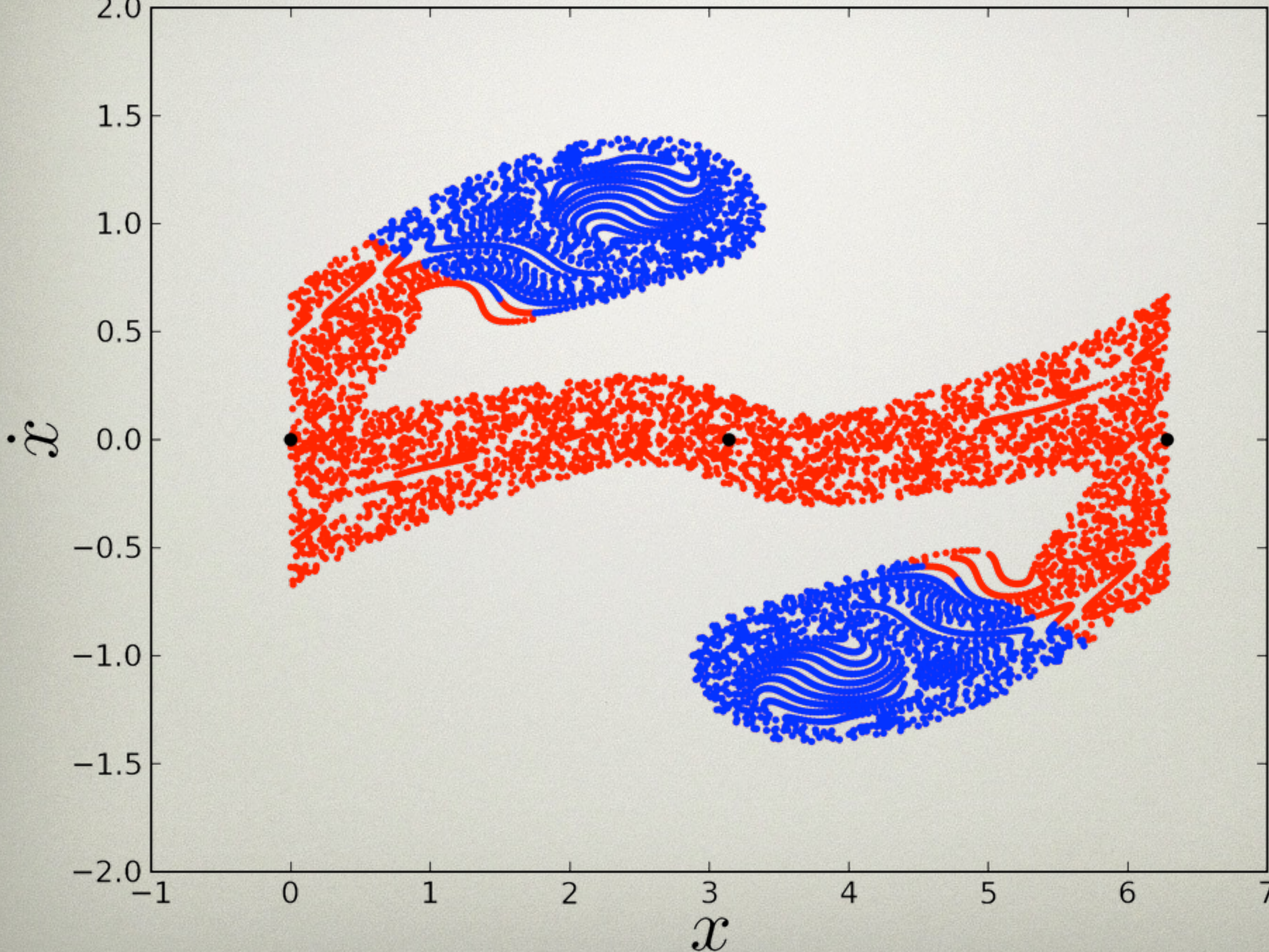


T^1
3
2
1
0
−1
−2
−3
−1
0
1
2
3
4
5
6
7
$\dot{x}$
x

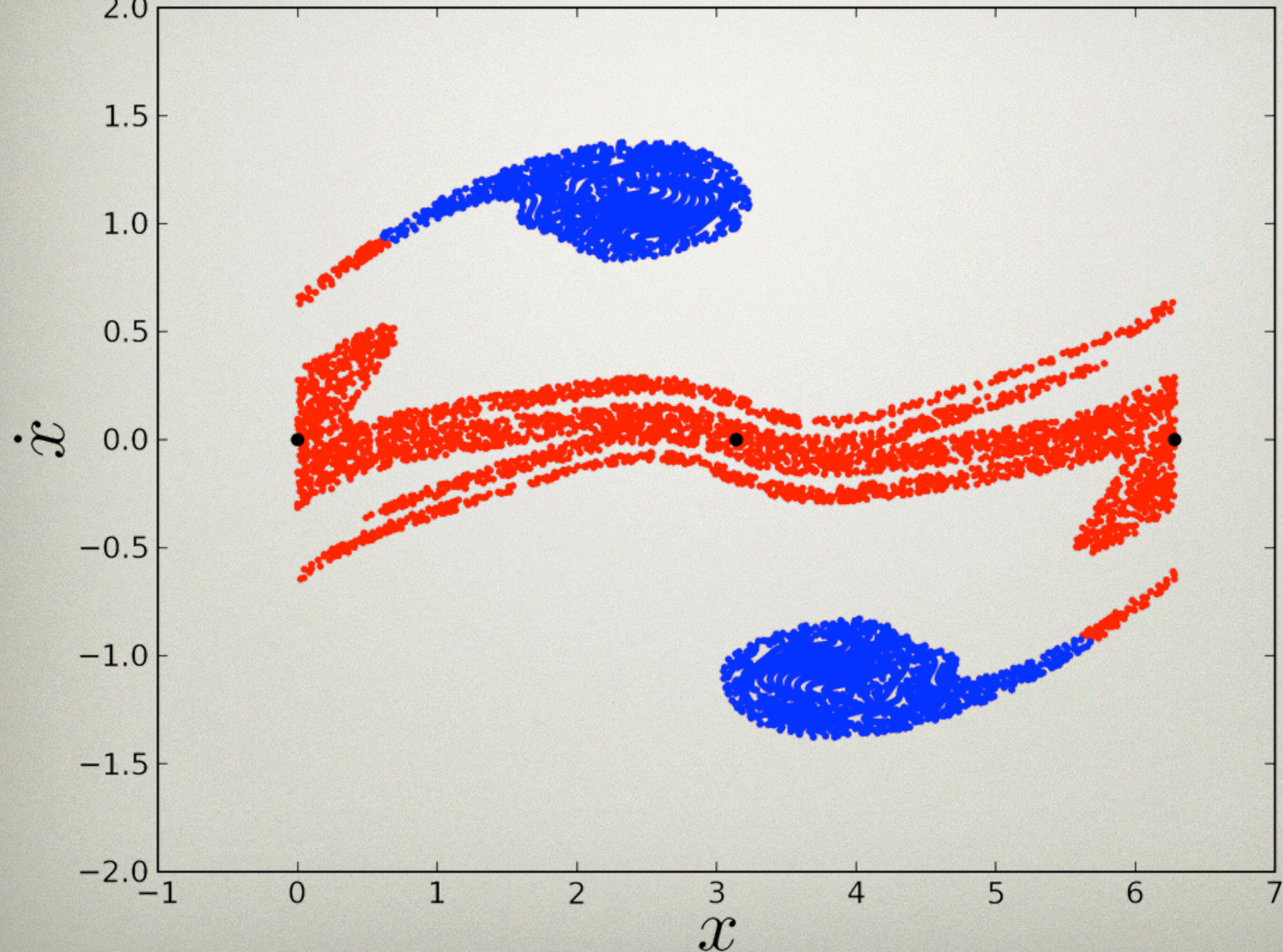


T^2
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−1
0
1
2
3
4
5
6
7
ẋ
x

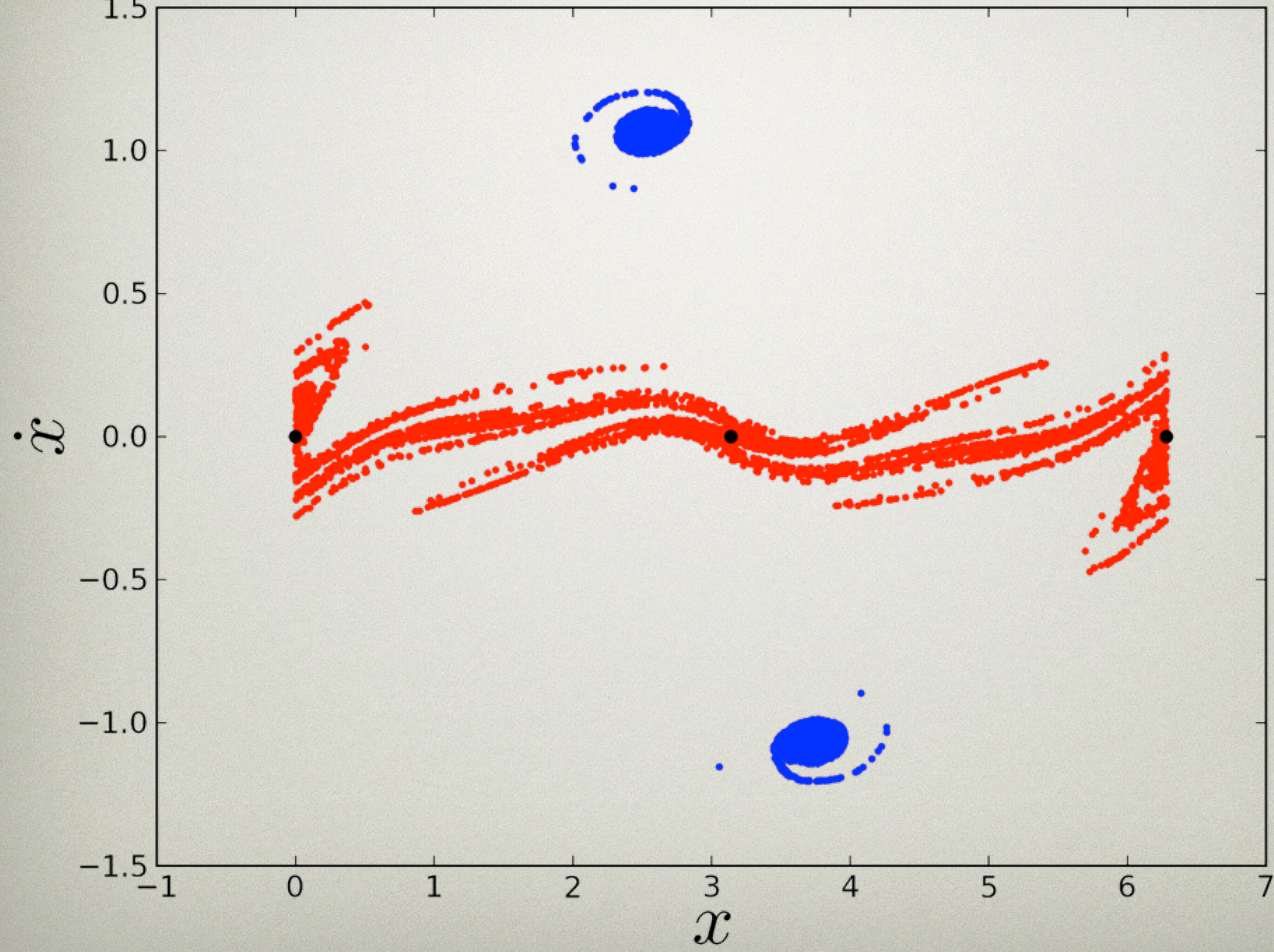
T^3



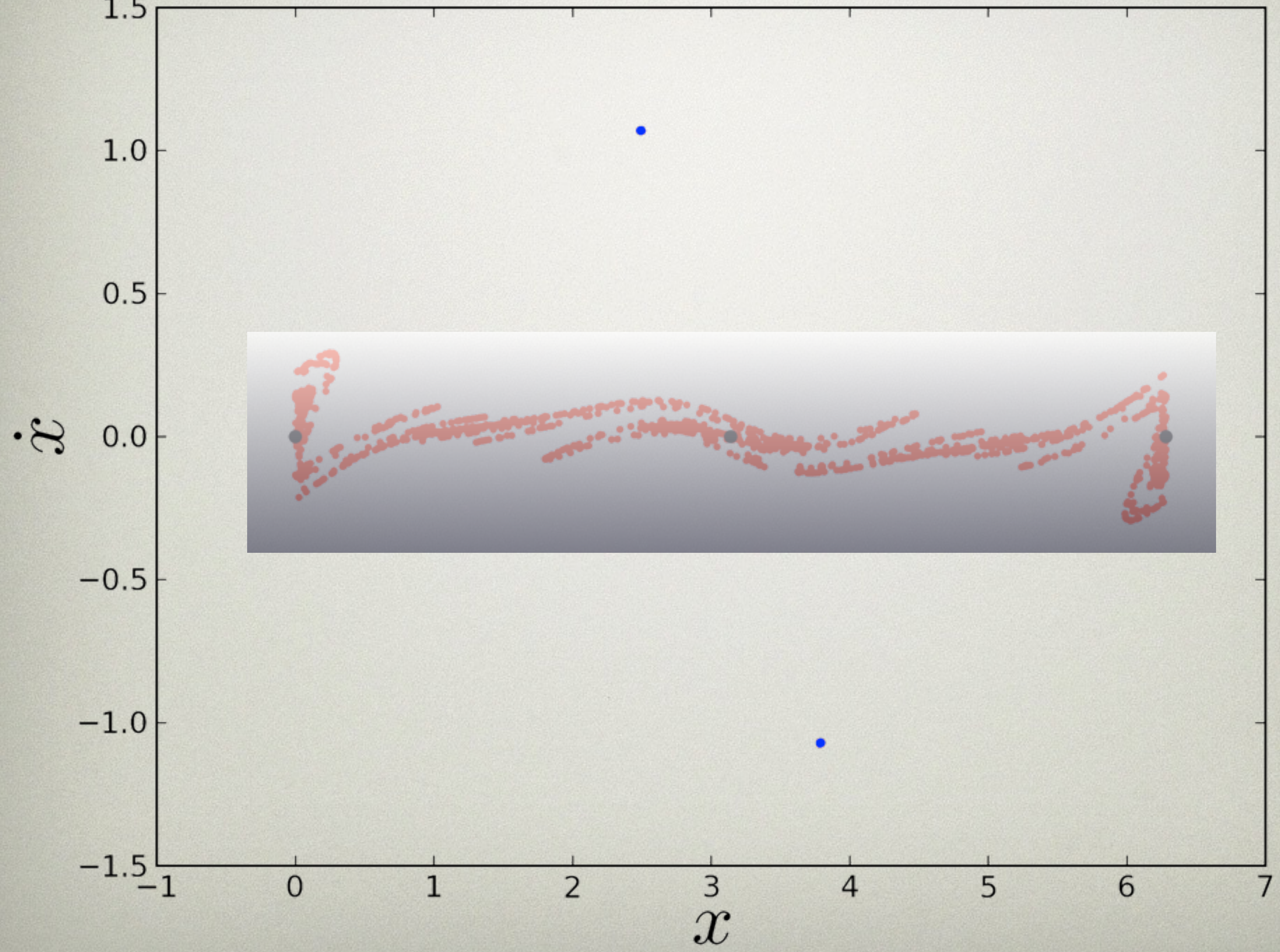
T^4

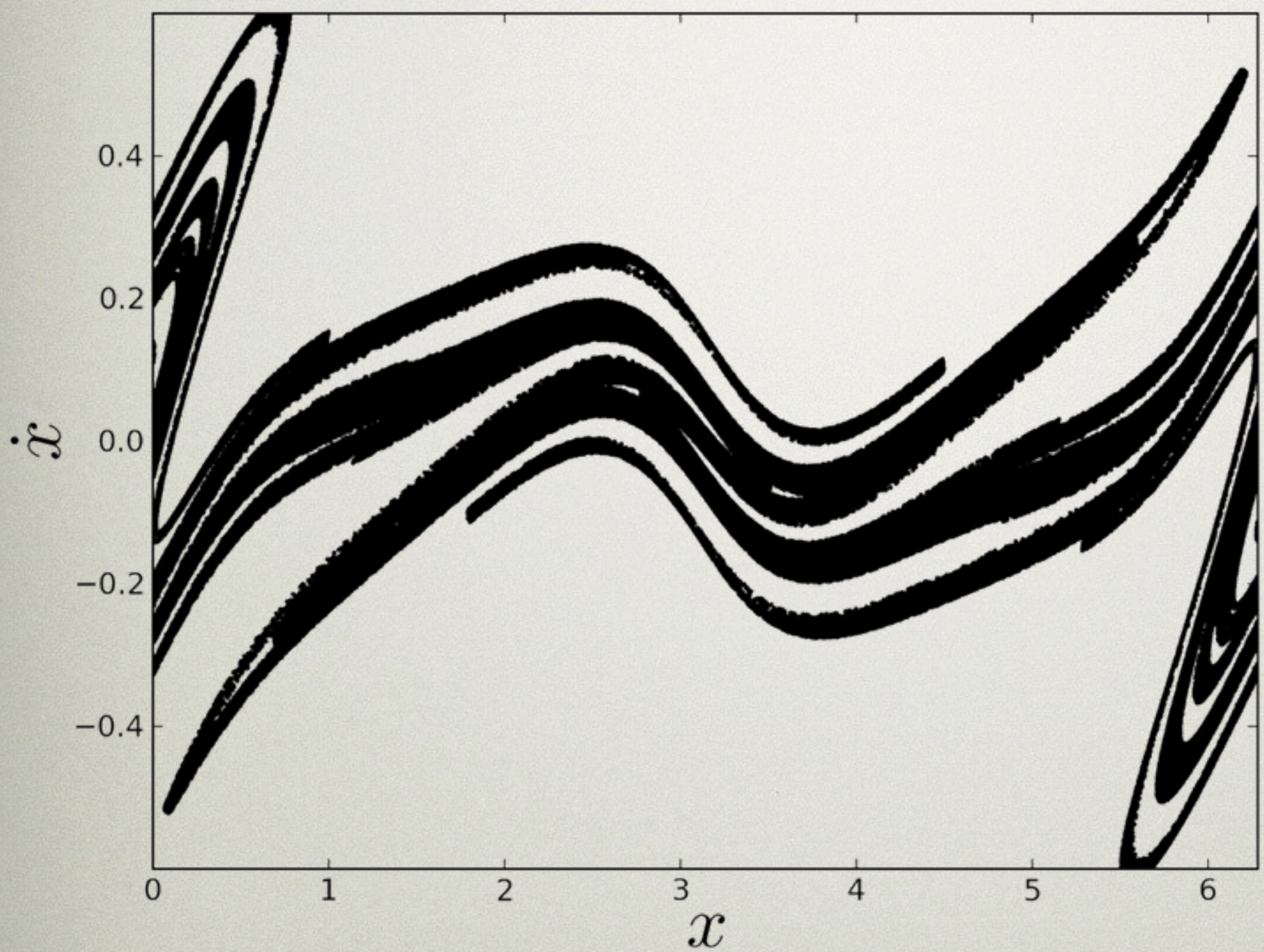


T^8



T^{21}



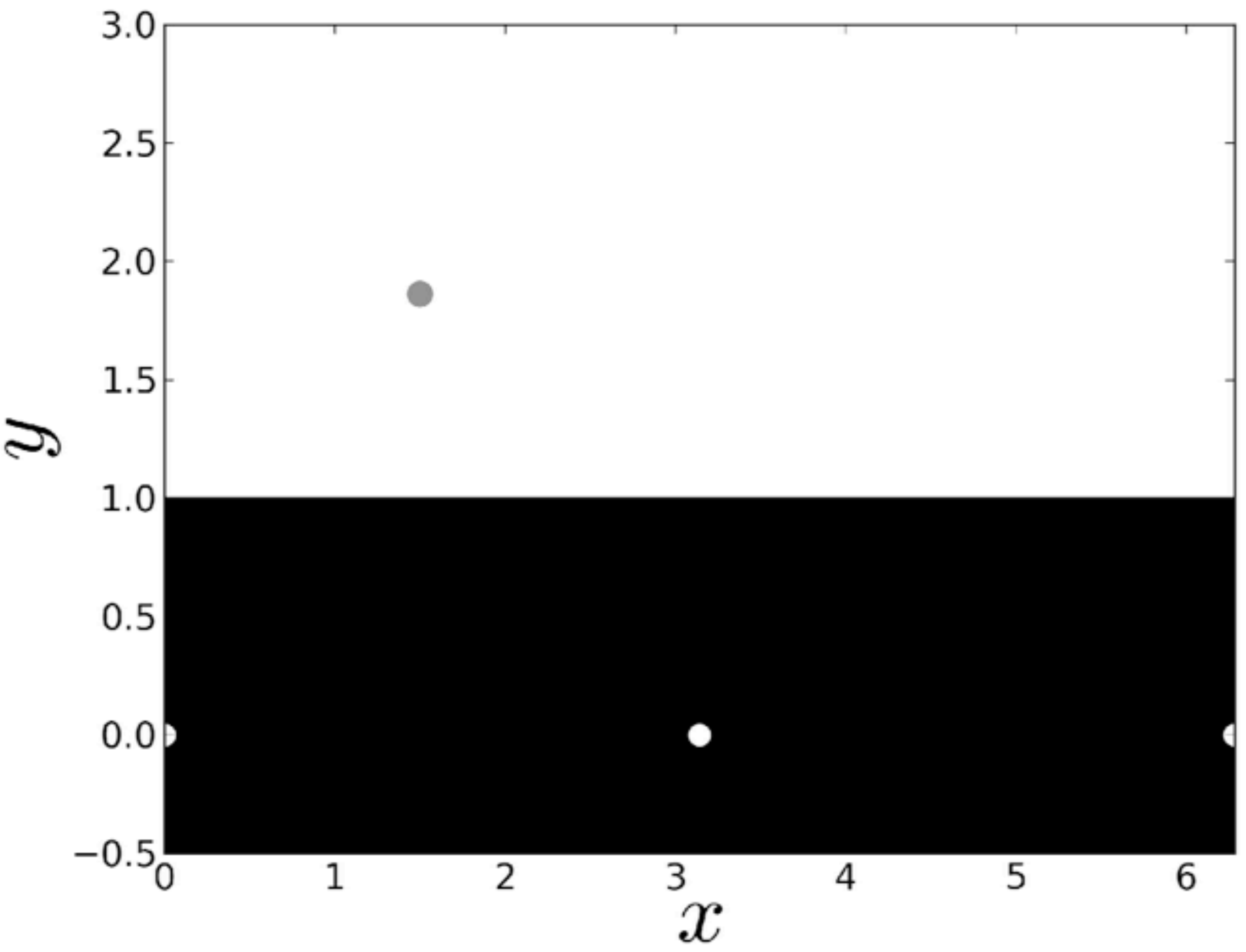


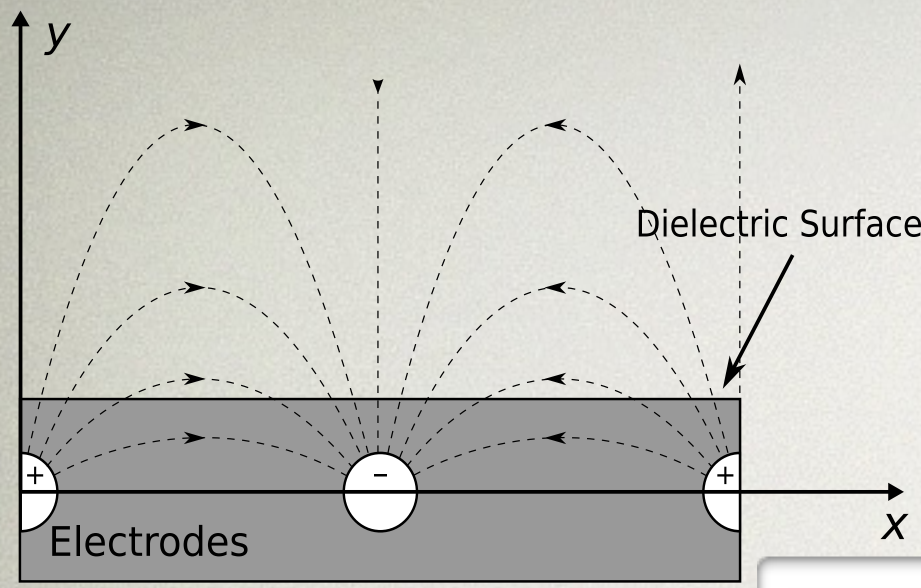
$\beta = 0.1$

$A = 0.3$

Lyapunov exponent: 0.165

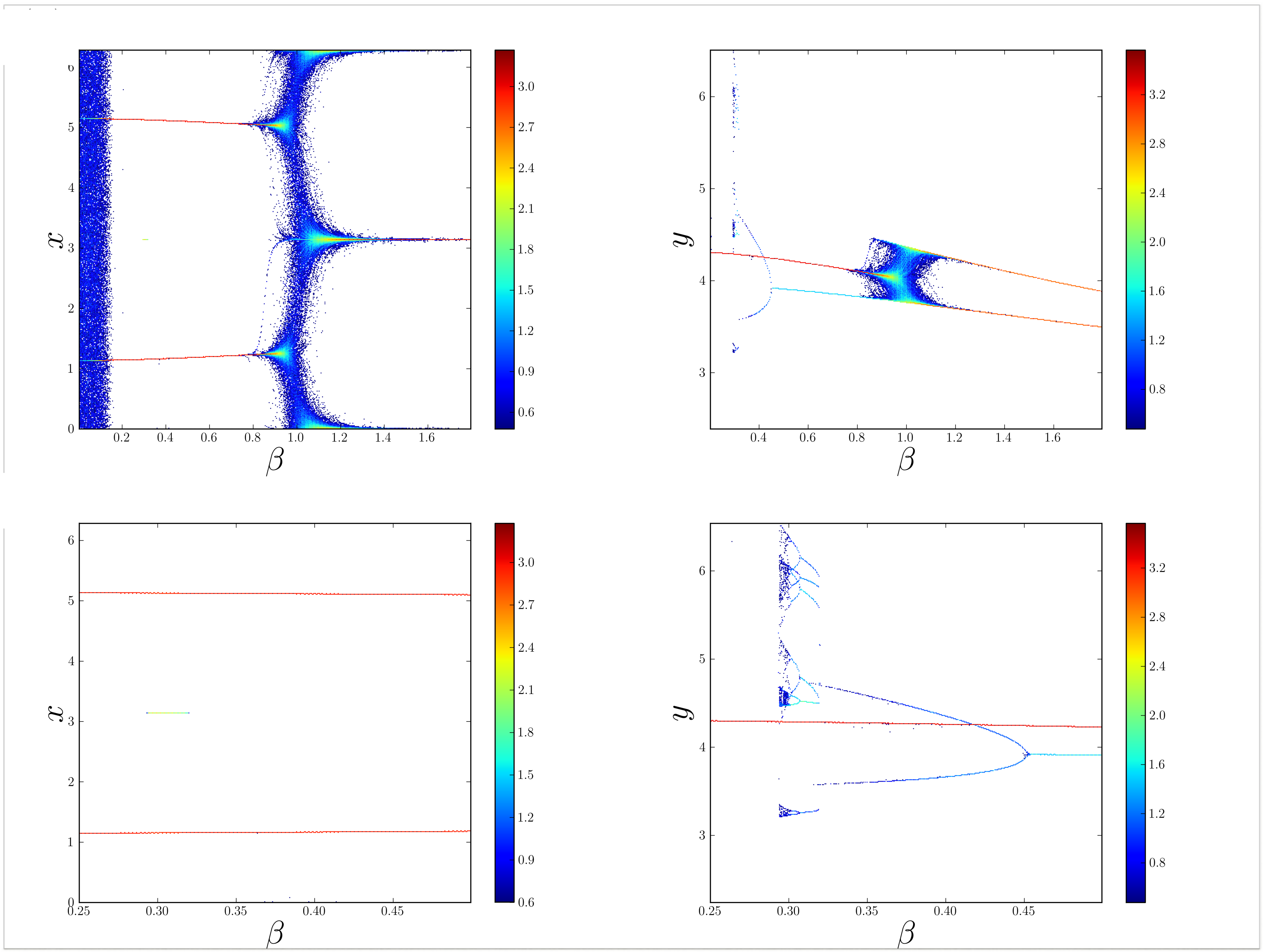
2D





Variations in the damping coefficient

A=9.0



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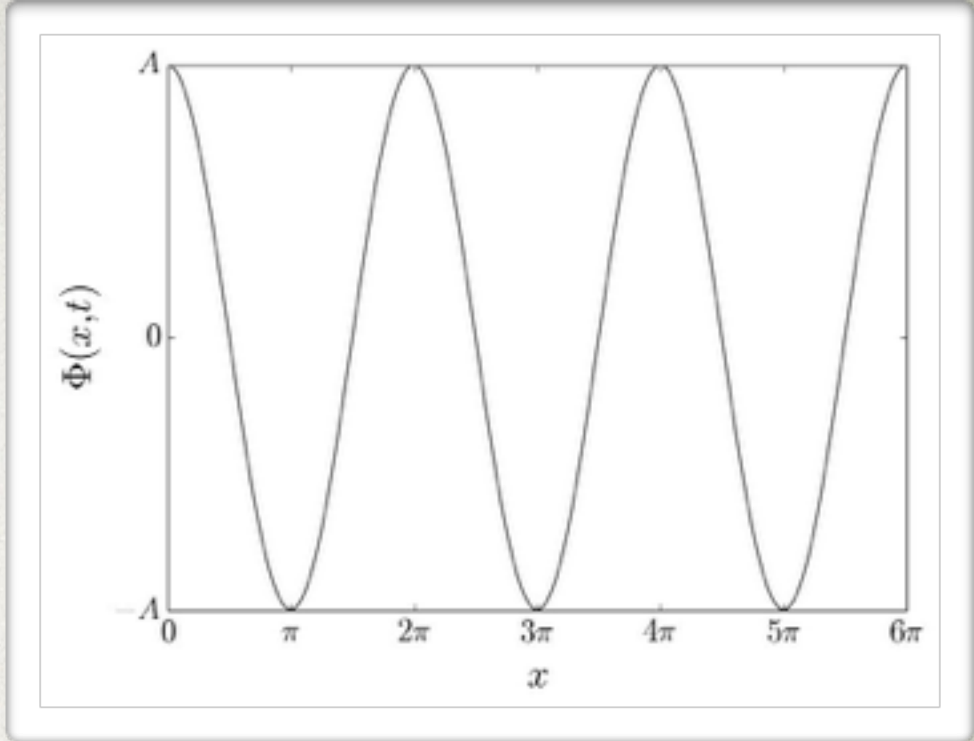
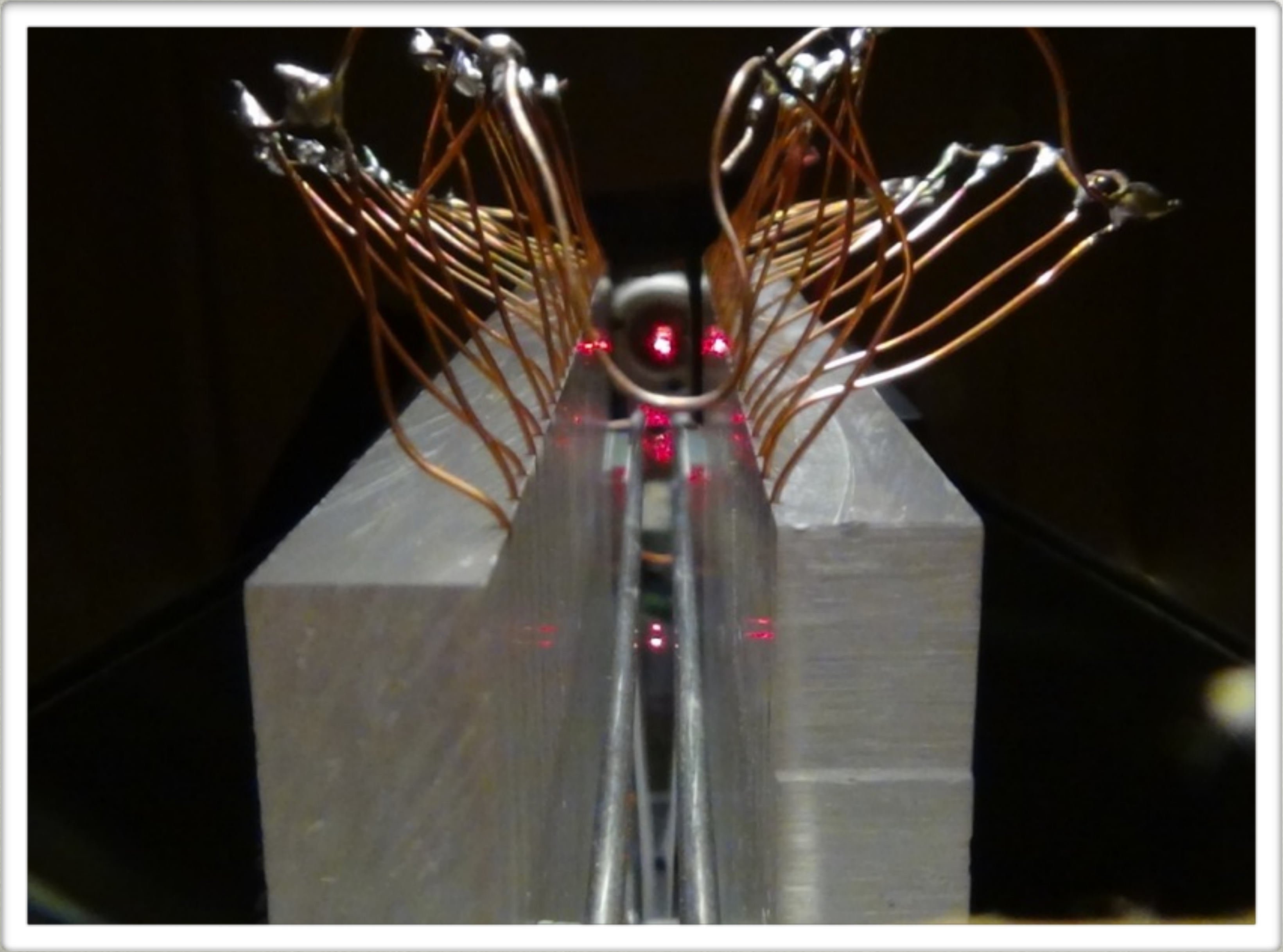




What do we mean by STP?

$\Phi(x,t) \equiv$ **The potential Energy**

$$\Phi(x,t) = f(x)g(t)$$

Periodic in time

Periodic in space

- The kicked rotor
  F. L. Moore, J. C Robinson, C. F. Bharucha, Bala Sundaram, M. G. Raizen (PRL 1995)

- Driven Josephson Junctions,
  E. Boukobza, M. G. Moore, D. Cohen, A. Vardi (PRL, 2010)

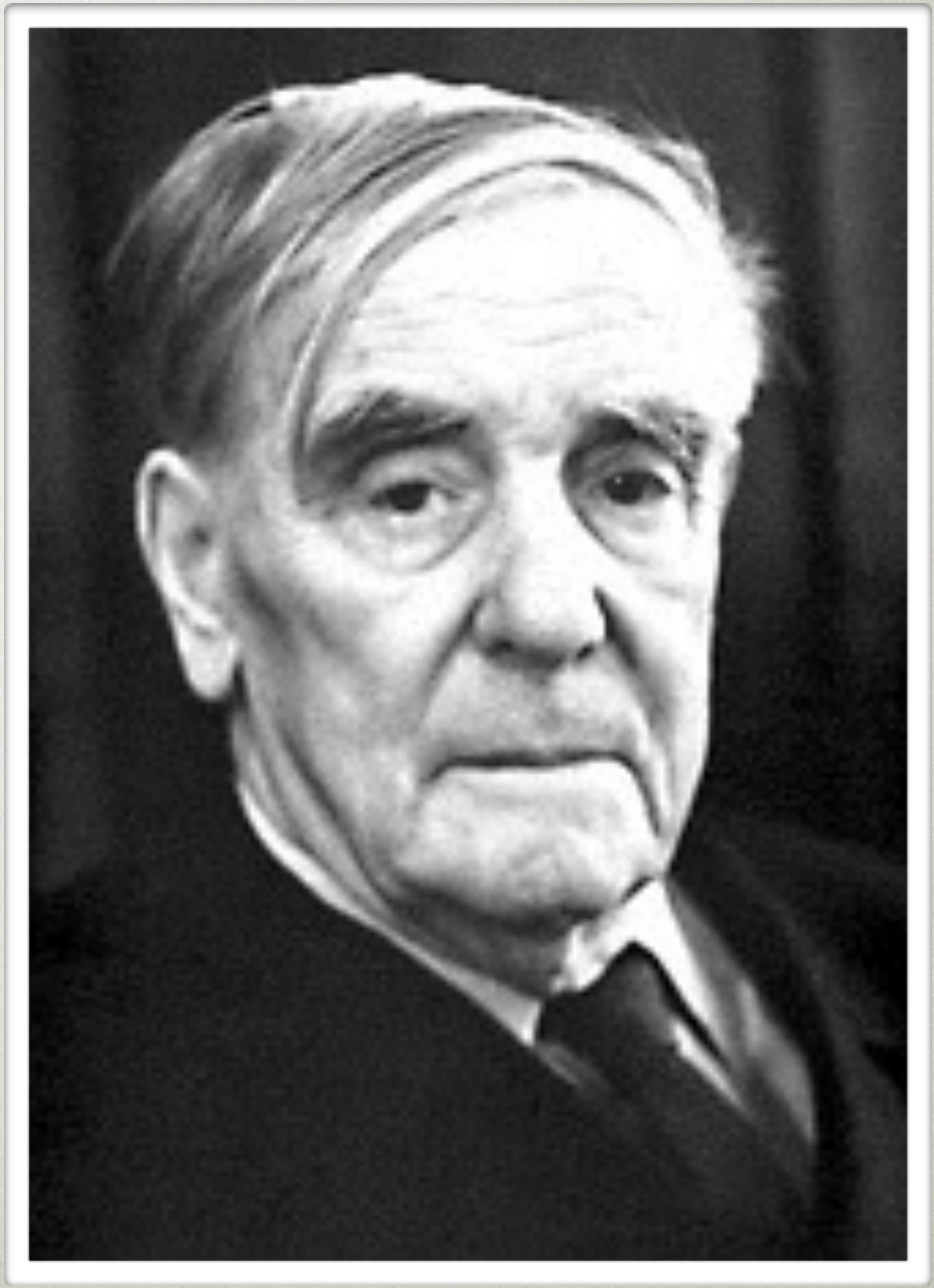
- Transport control & ratchets

  - Hamiltonian, H. Schanz, M. F. Otto, R. Ketzmerick, T. Dittrich (PRL,2001)

  - Damped, Jose L. Mateos (PRL, 2000)

- Dynamic stabilization and potential renormalization,
  A. Wickenbrock, P. C. Holz, N. A. Abdul Wahab, P. Phoonthong, D. Cubero, F. Renzoni(PRL, 2012)

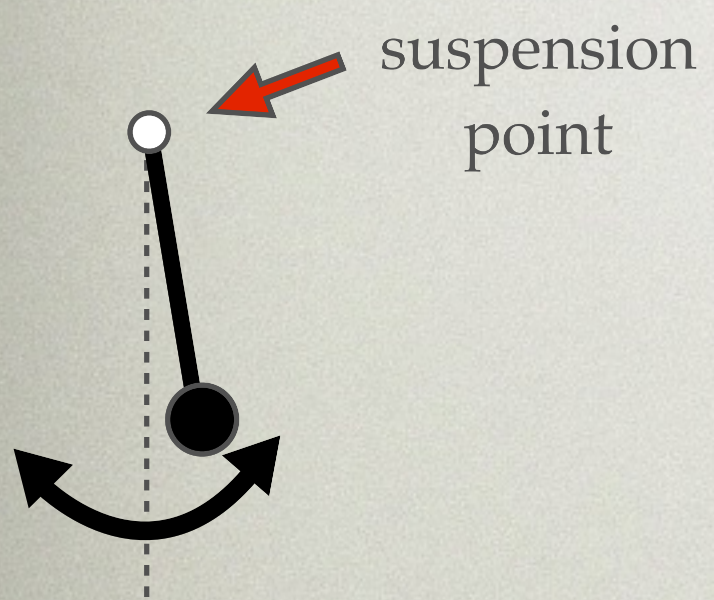
Pyotr Kapitza
1894-1984



Example: Kapitza's pendulum

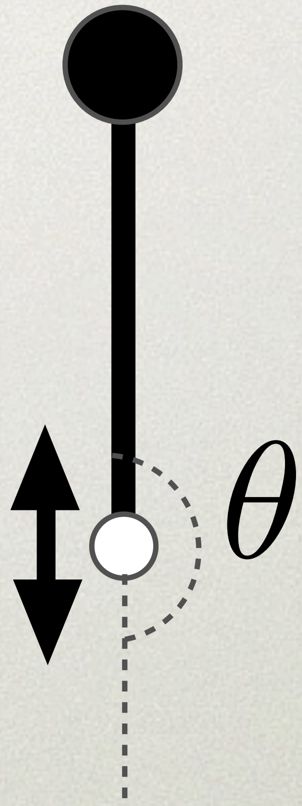
Kapitza's Pendulum

Pendulum



case 1

case 2



$$\ddot{\theta} + \beta\dot{\theta} + \left(g - a\omega^2 \cos \omega t\right) \sin \theta = 0$$

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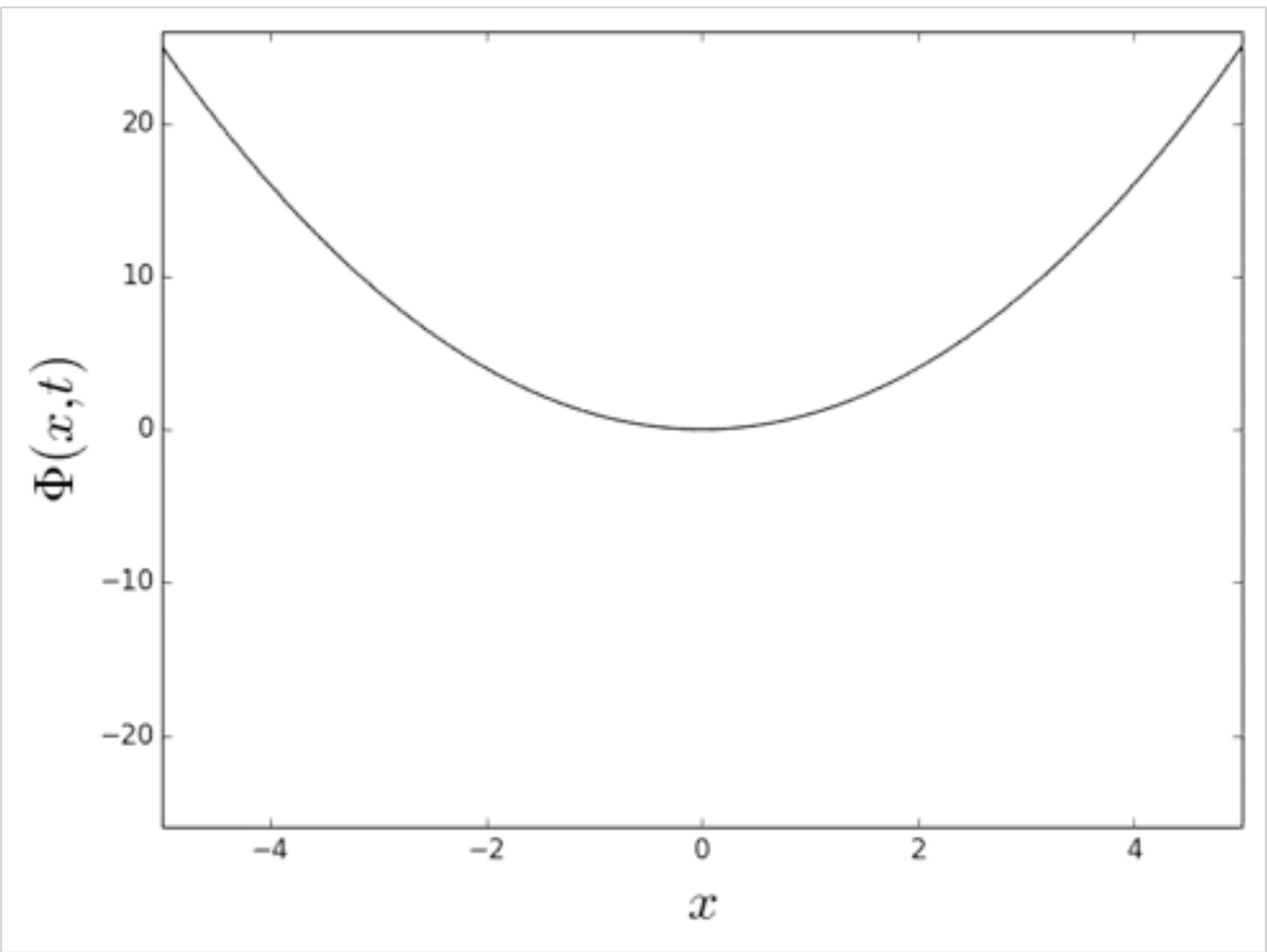
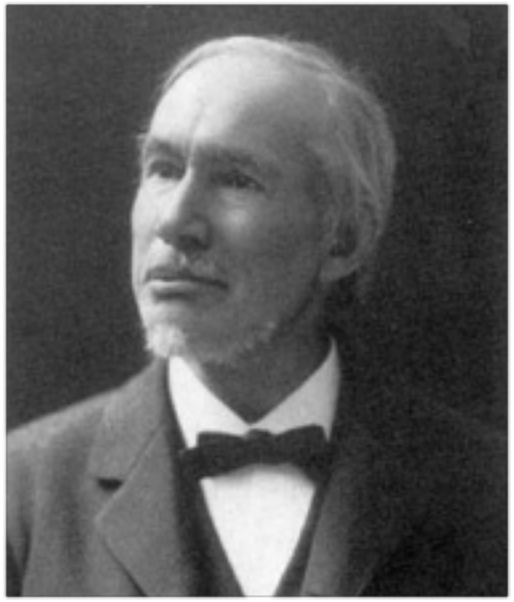
Linearize the equations of motion

$$\Phi = g(t)x^2 \Rightarrow \ddot{x} + g(t)x = 0$$



The Hill Equation



Hill, "On the Part of the Motion of Lunar Perigee Which is a Function of the Mean Motions of the Sun and Moon."
Acta Math. 8, 1-36, 1886.

The simple first order case of the Hill equation:

$$g(t) = \cos \omega t$$

$$\ddot{x} + (a - 2q\cos 2t)x = 0$$

The Mathieu equation

Solutions:

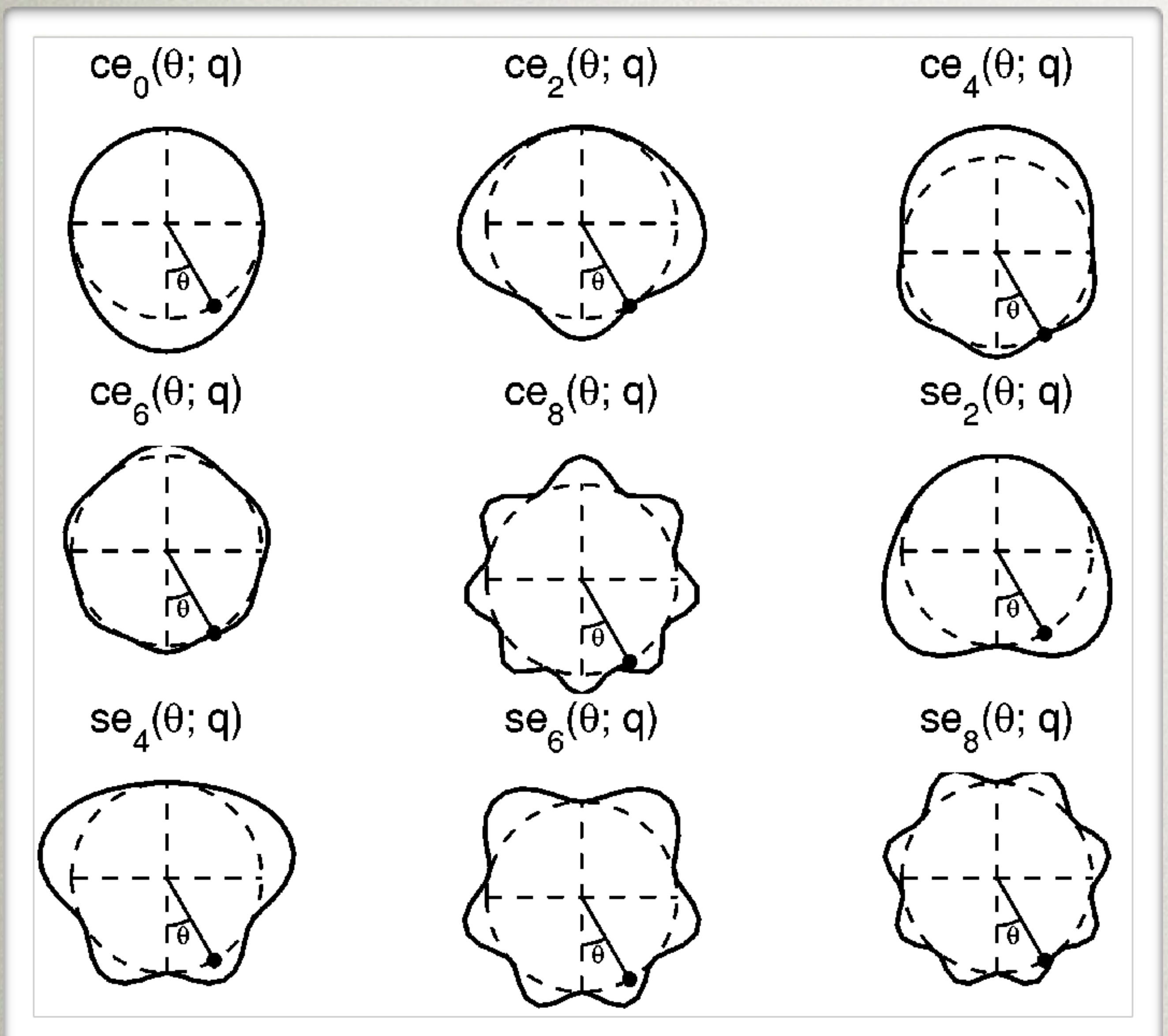
$$\mathrm{ce}_m(t)$$

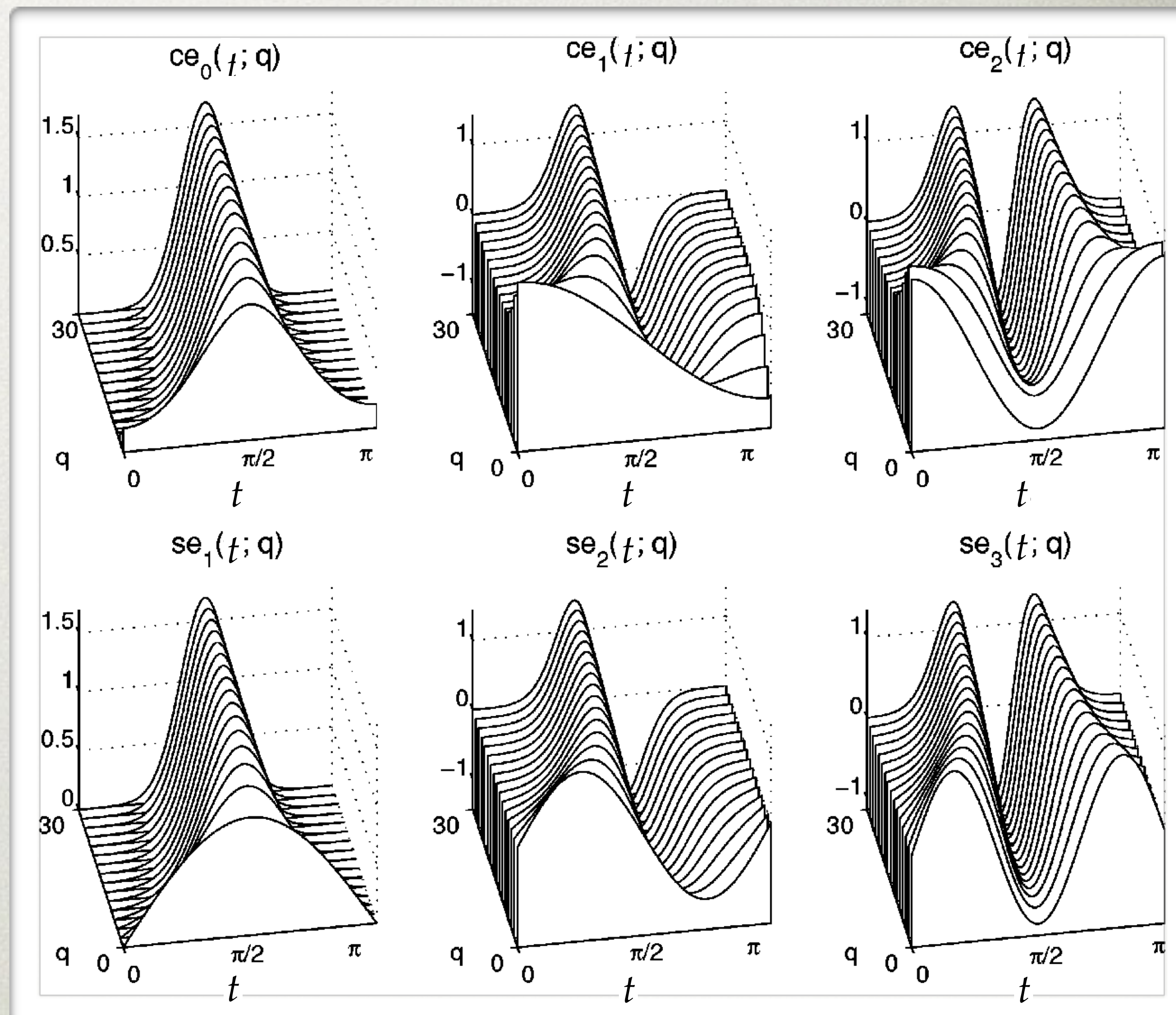
cosine-elliptic

$$\mathrm{se}_m(t)$$

sine-elliptic

$$\ddot{x} + (a - 2q\cos 2t)x = 0$$

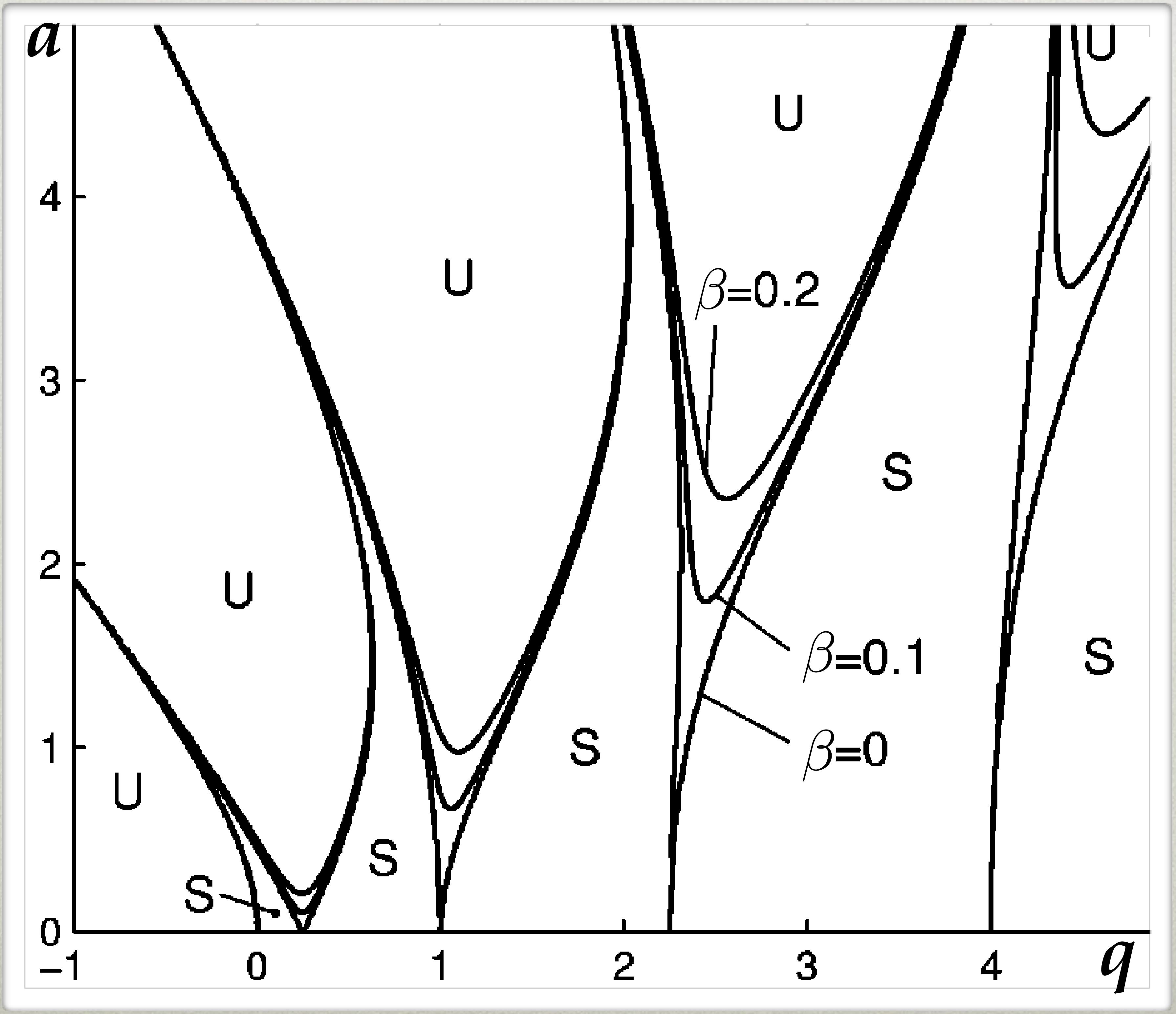




Gutierrez-Vega, Rodrıguez-Dagnino, Meneses-Nava, Cha´vez-Cerda,

“Mathieu functions, a visual approach,” (2002)

Stability Regions



Another Approach:

- **Krylov-Bogoliubov averaging method**

Effective Liouville equation for classical driven system (arXiv:cond-mat/9806137v2 [cond-mat.stat-mech])

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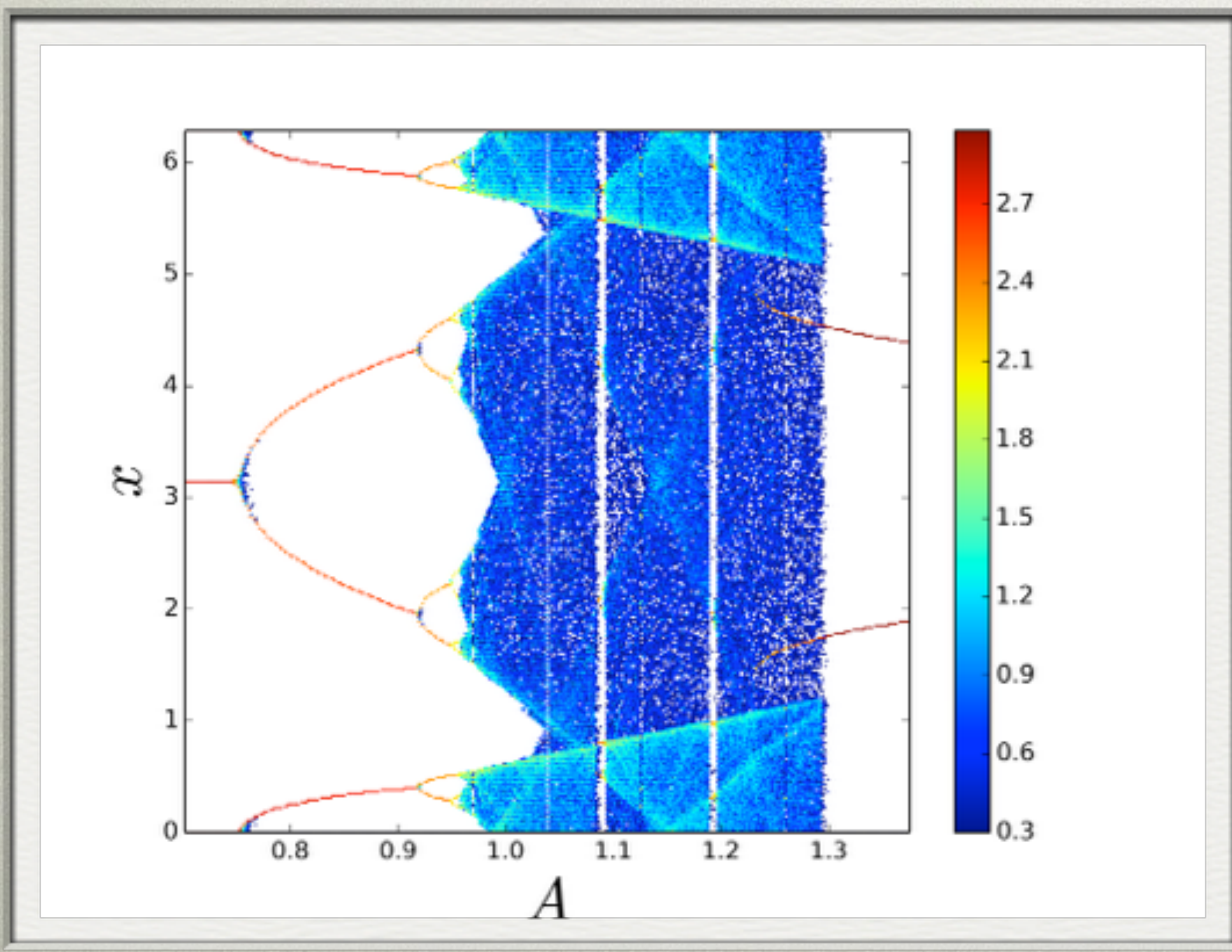
Model Potential

$$\Phi(x,t) = -A\cos x \cos t$$

$\lambda = 2\,\pi$

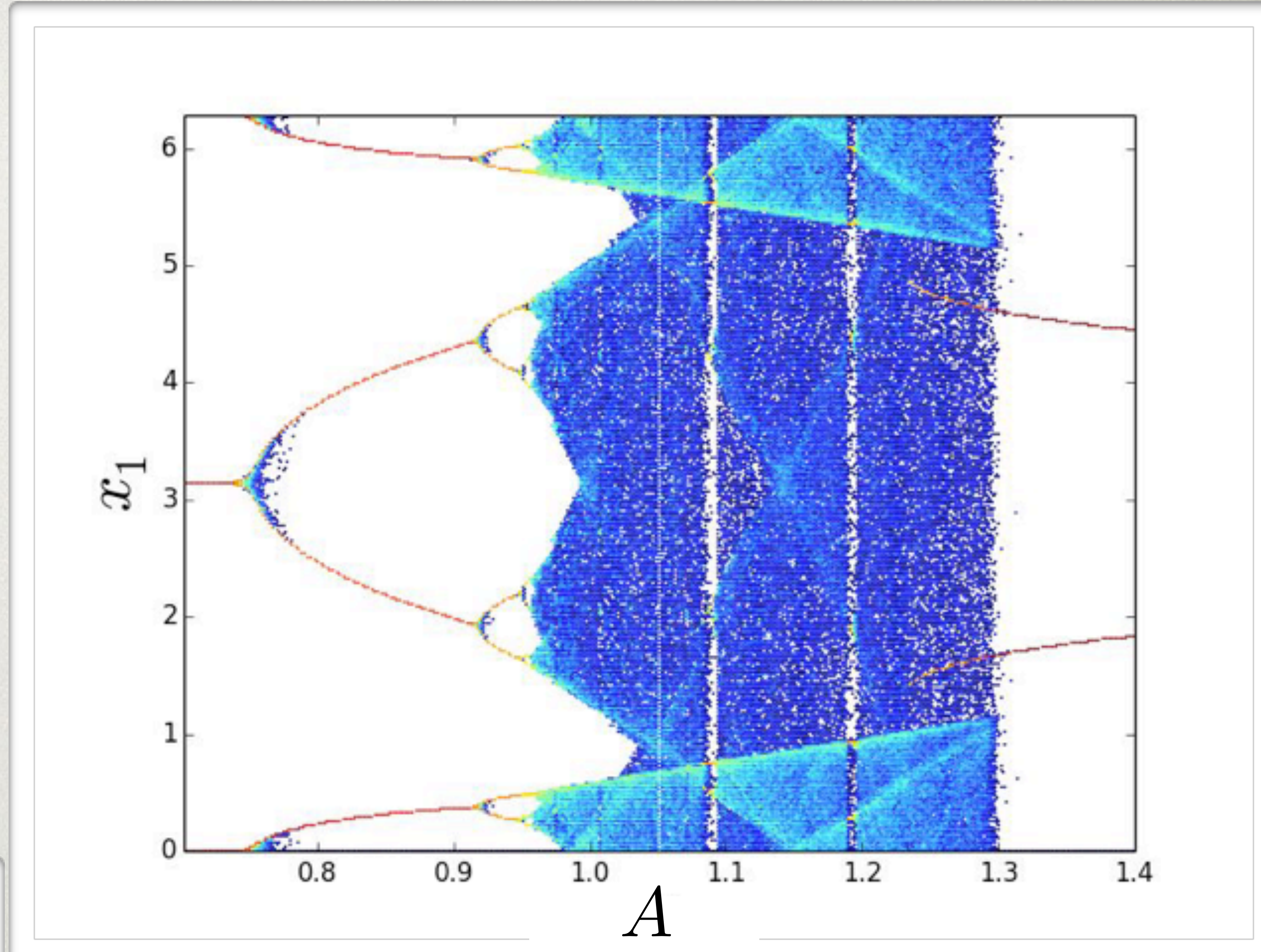


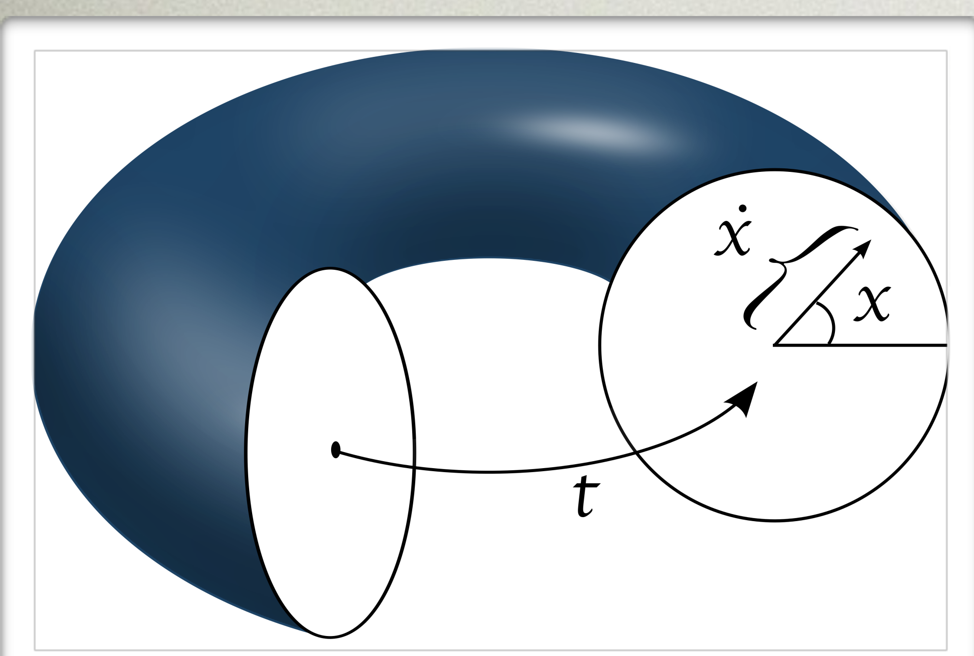
Equation of motion

$$\ddot{x} = A\sin x \cos t - \beta\dot{x}$$

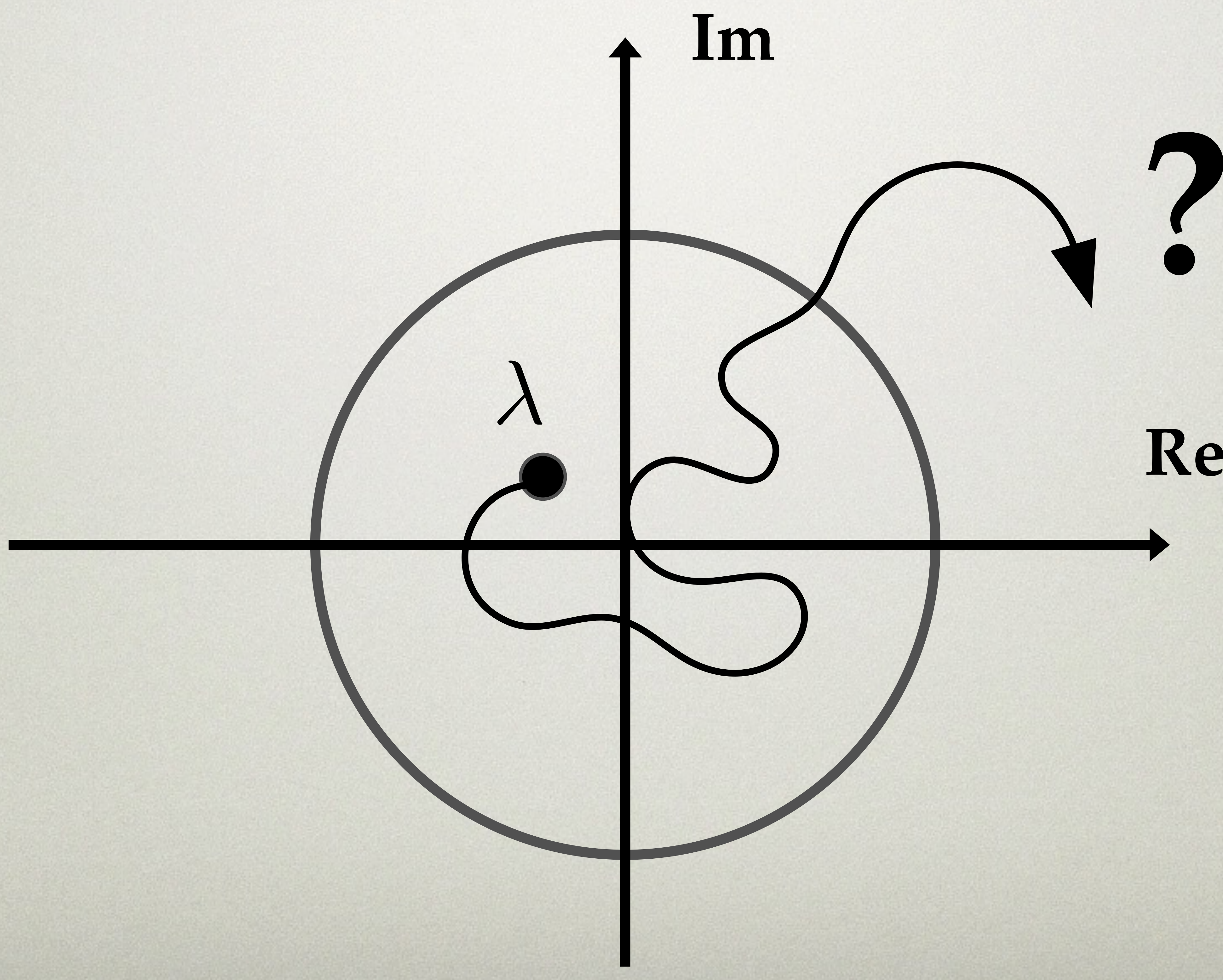
$$\beta = 0.6$$

A variety of choices of Poincaré

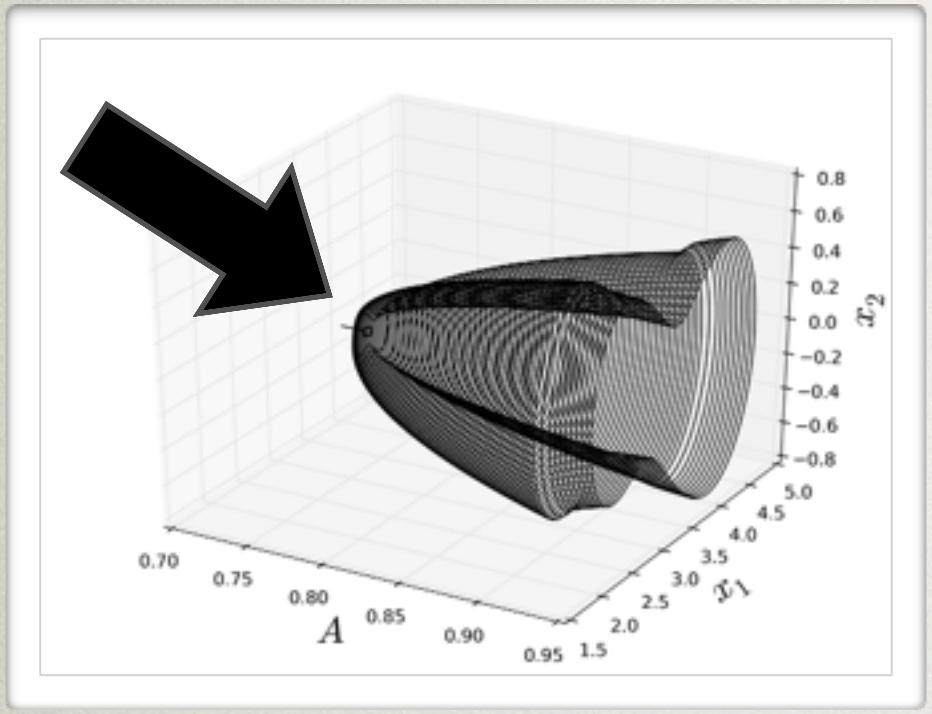


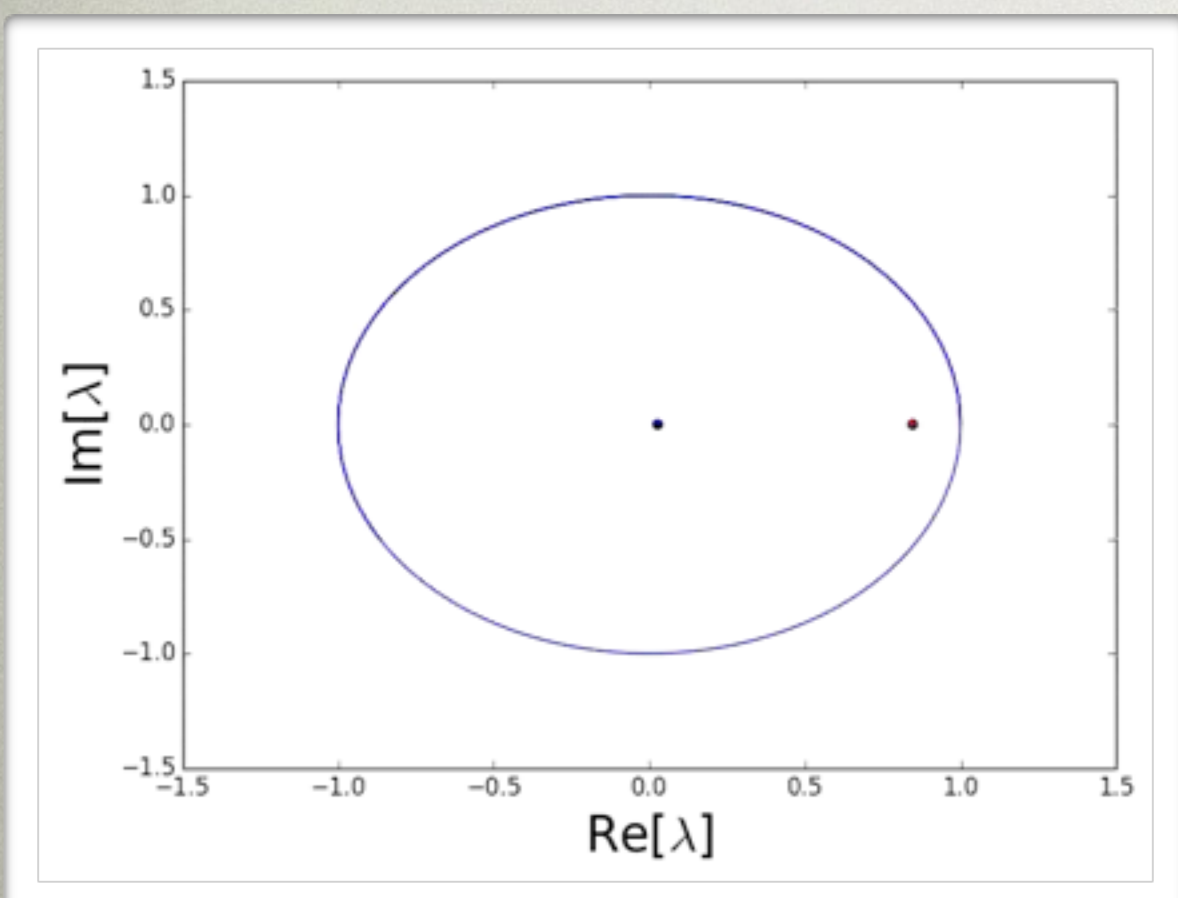


Bifurcations

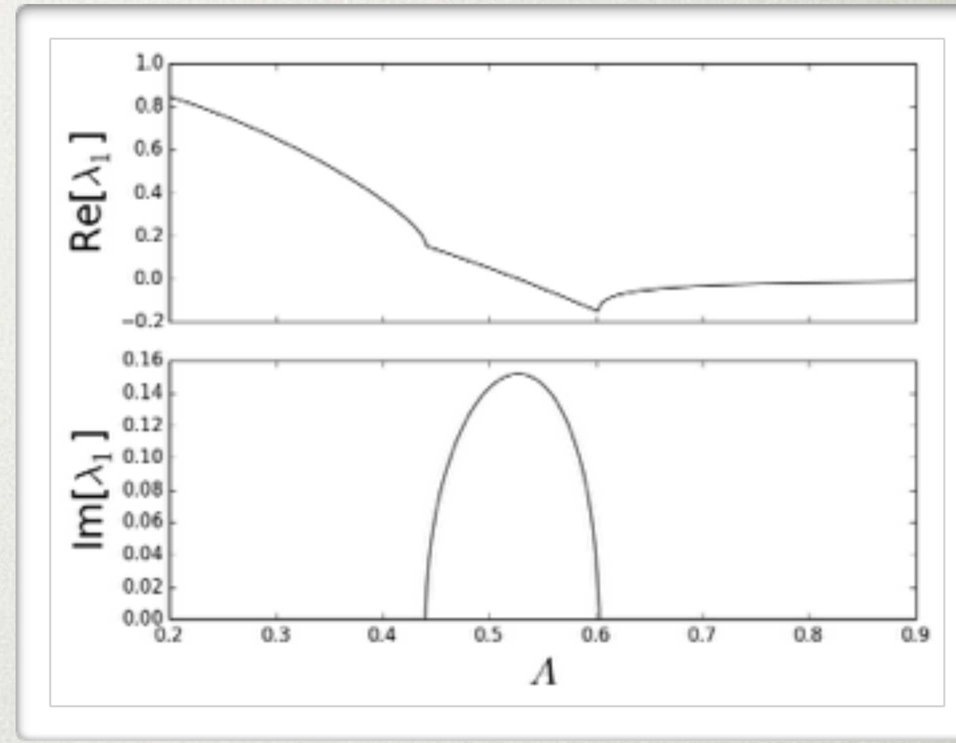


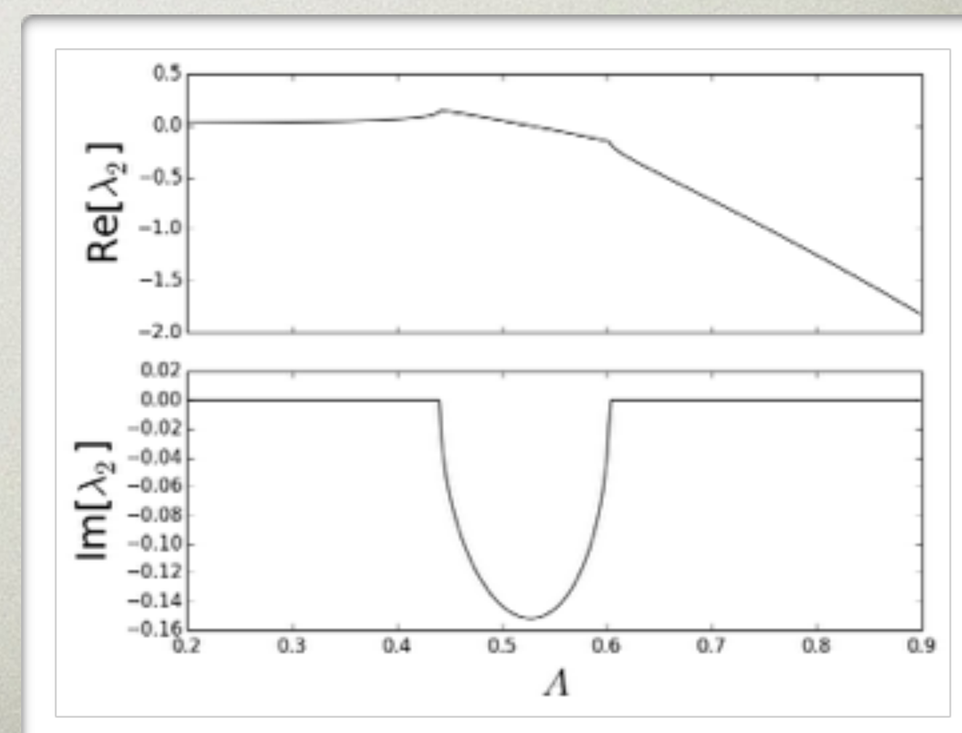
Why is this not a Hopf Bifurcation?





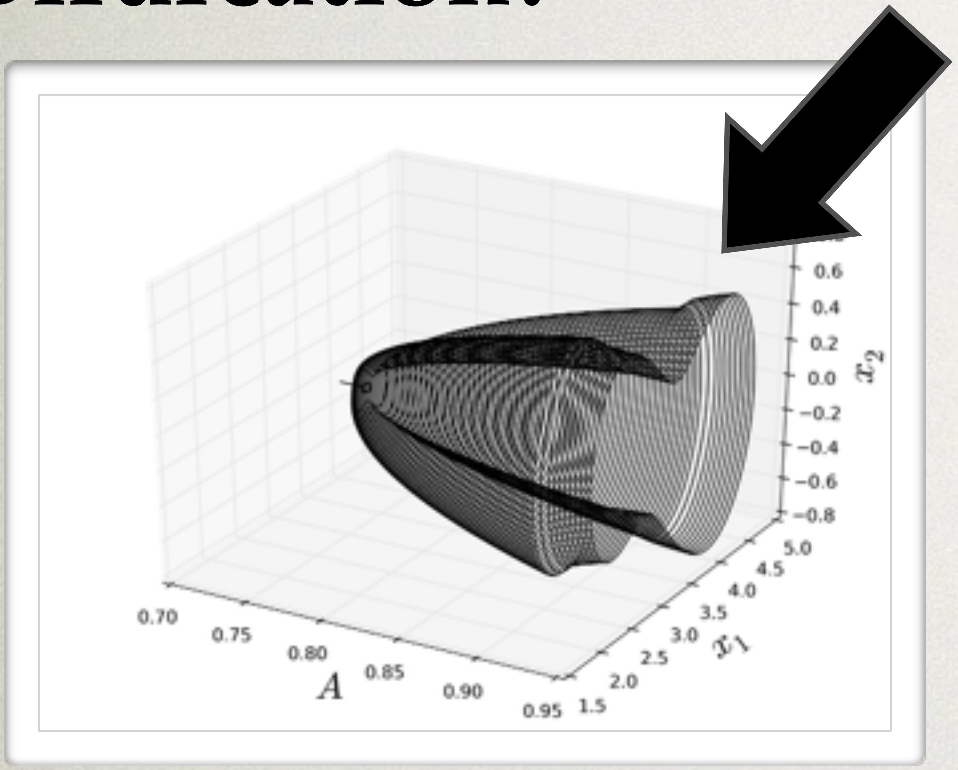
Lets look at the stability multipliers

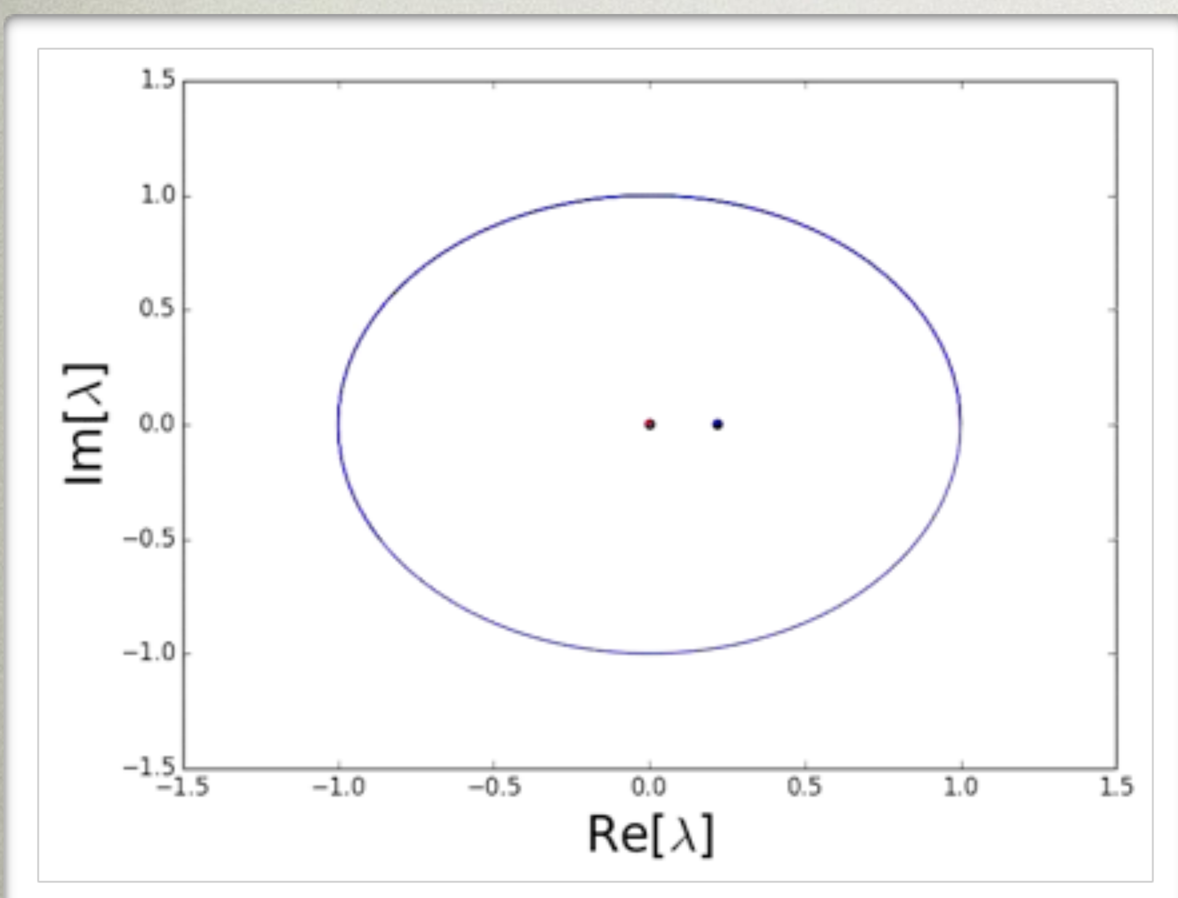


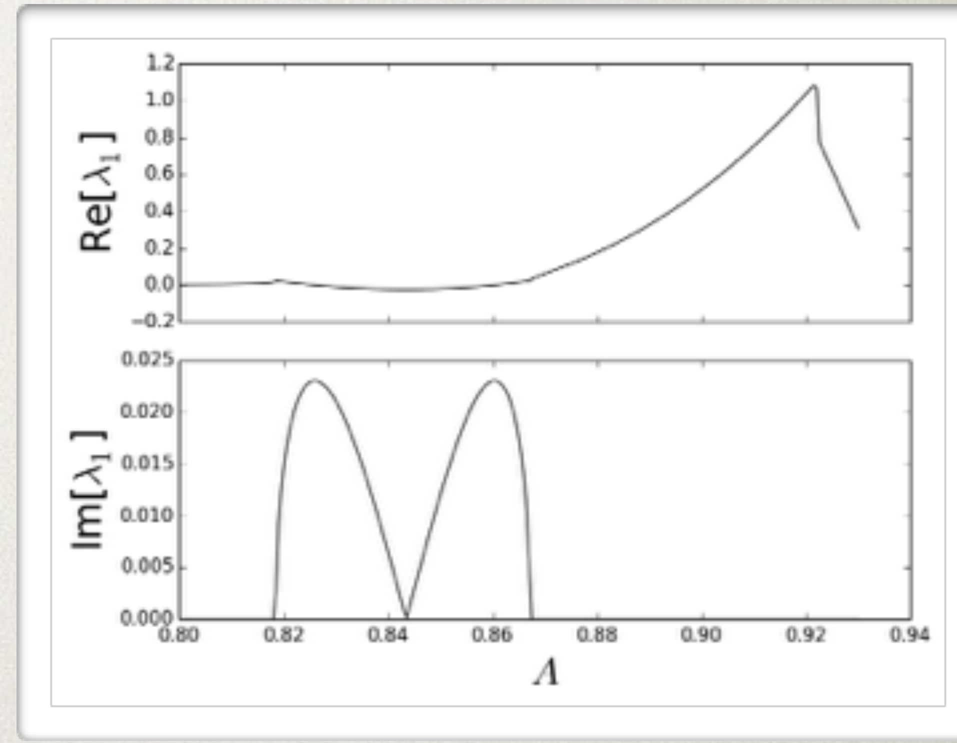


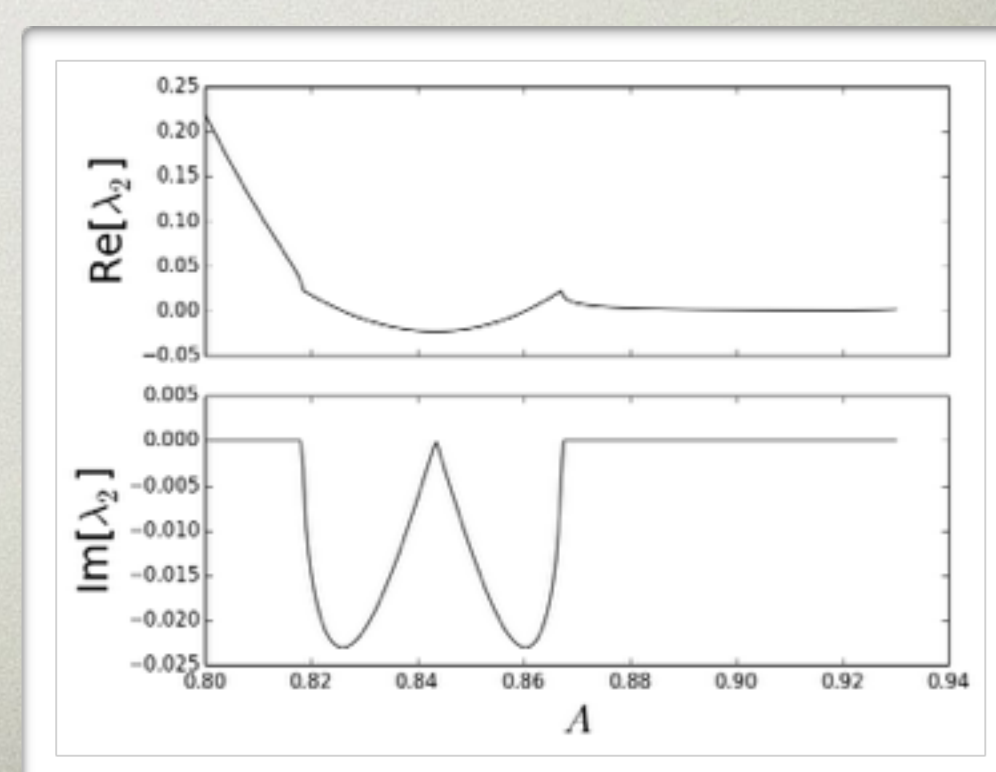
The next Bifurcation?

Lets look at the stability multipliers

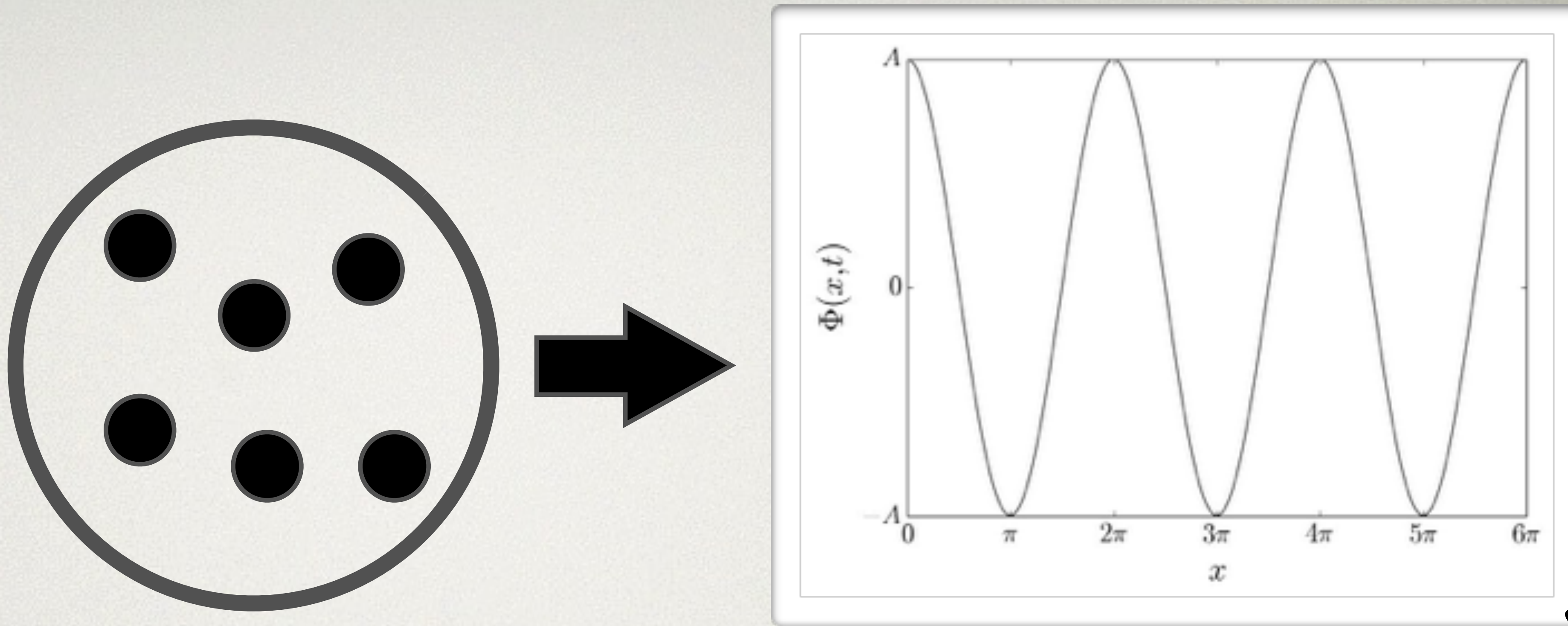








Multiple Particles



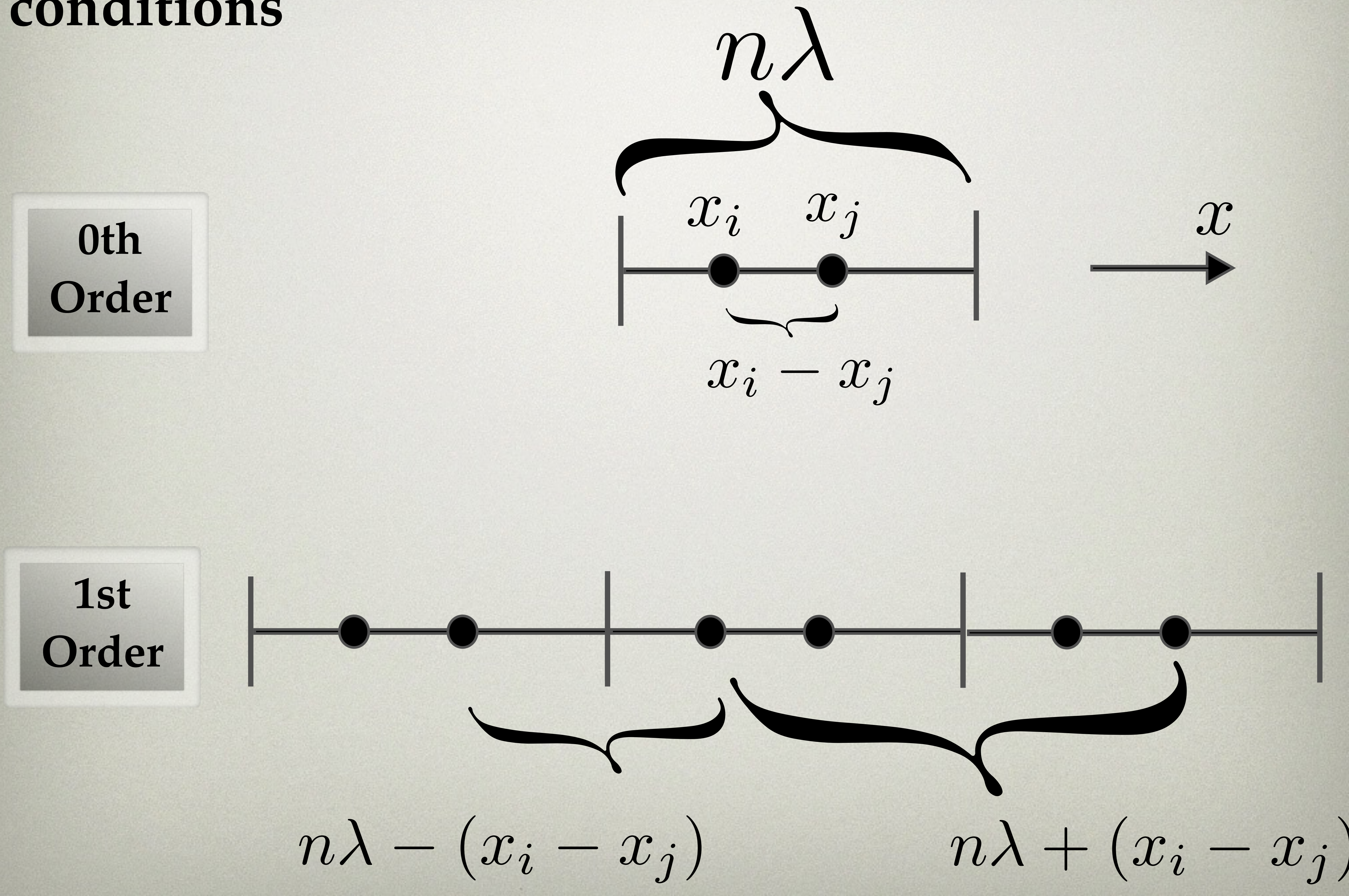
$n\lambda$

concentration $= N/n$

$$\mathbf{F}_i = -\nabla\Phi_{\text{STP field}} - \nabla\Phi_{\text{interparticle}}$$

$$\Phi_{\text{interparticle}} = \frac{q^2}{r}$$

Long range interactions in periodic boundary conditions



$$\sum F_{\text{images}}$$

May be expressed in terms of polygamma function

$$F_{ij} = \frac{q^2 r_{ij}}{\|r_{ij}\|^3} - \left(\frac{q}{n\lambda}\right)^2 \left(\psi^{(1)}(-r_{ij}/\lambda) + \psi^{(1)}(r_{ij}/\lambda)\right)$$

Known to arbitrary precision

$q = 1$

$\beta = 0.6$

$m = 1$

$n = 1$

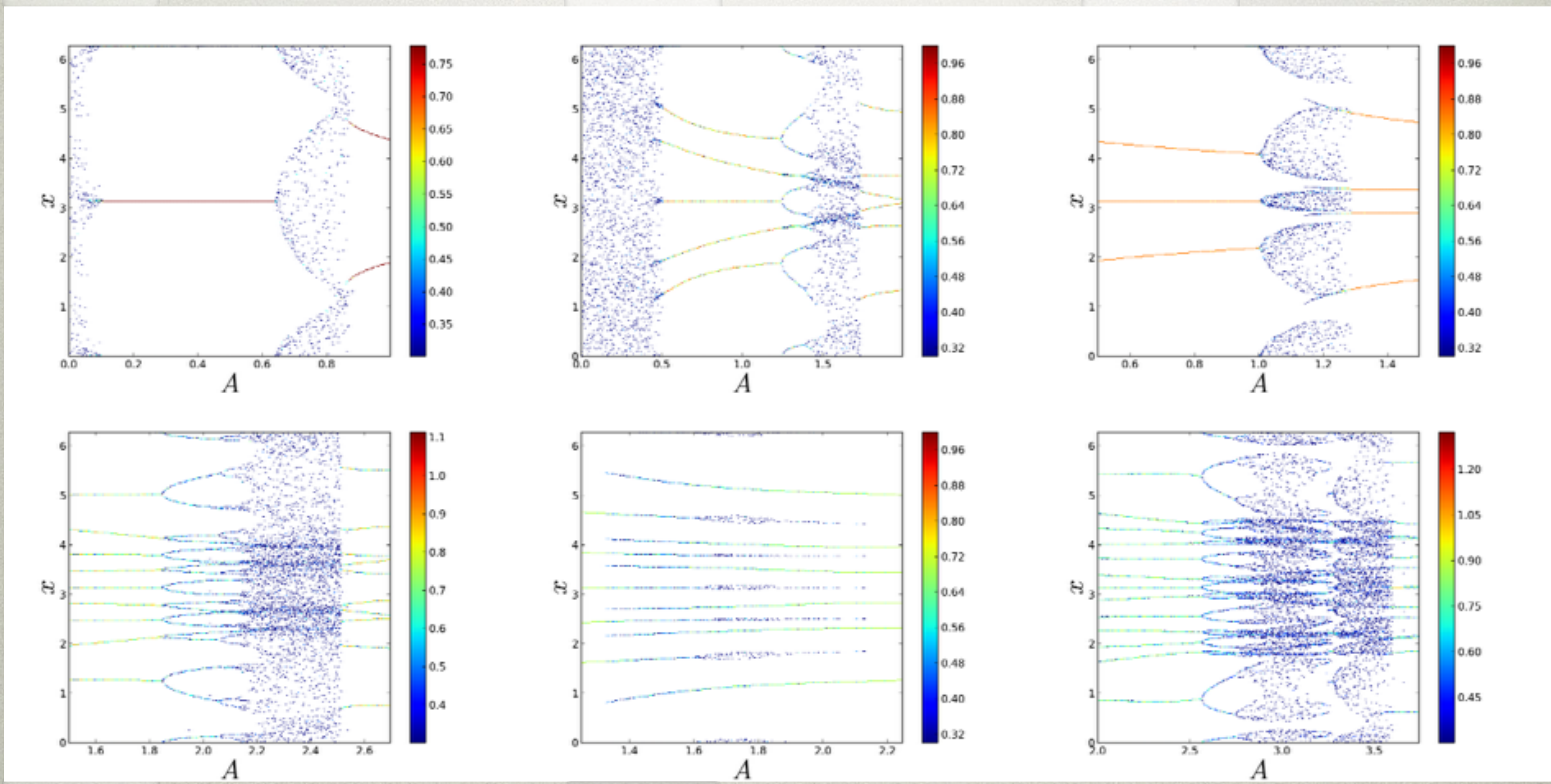
Integer concentrations

Multiple particle bifurcation diagrams

N=2

N=3

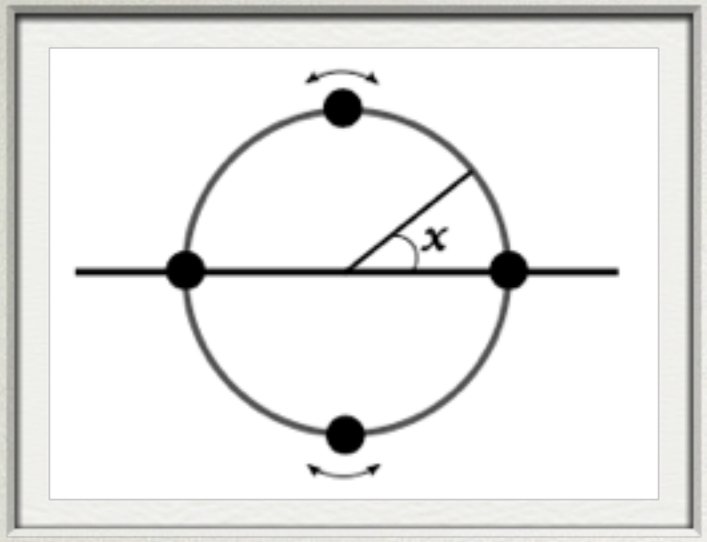
N=4



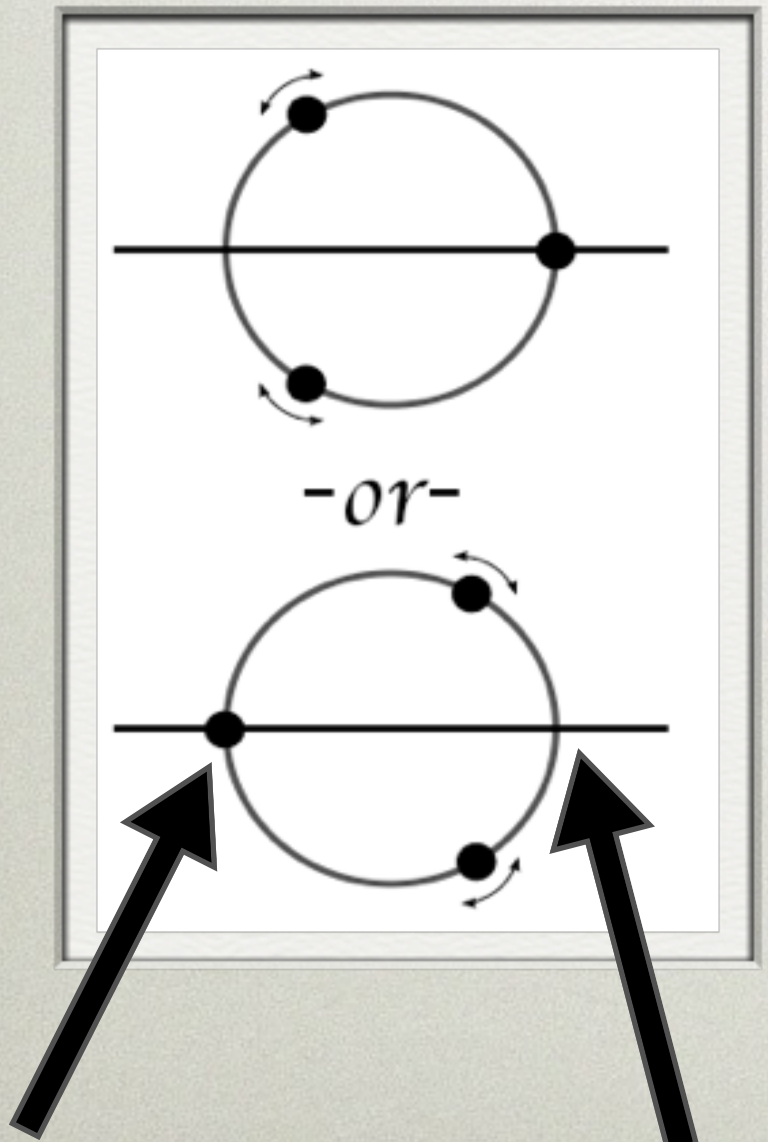
N=5

N=6

N=7



N=4



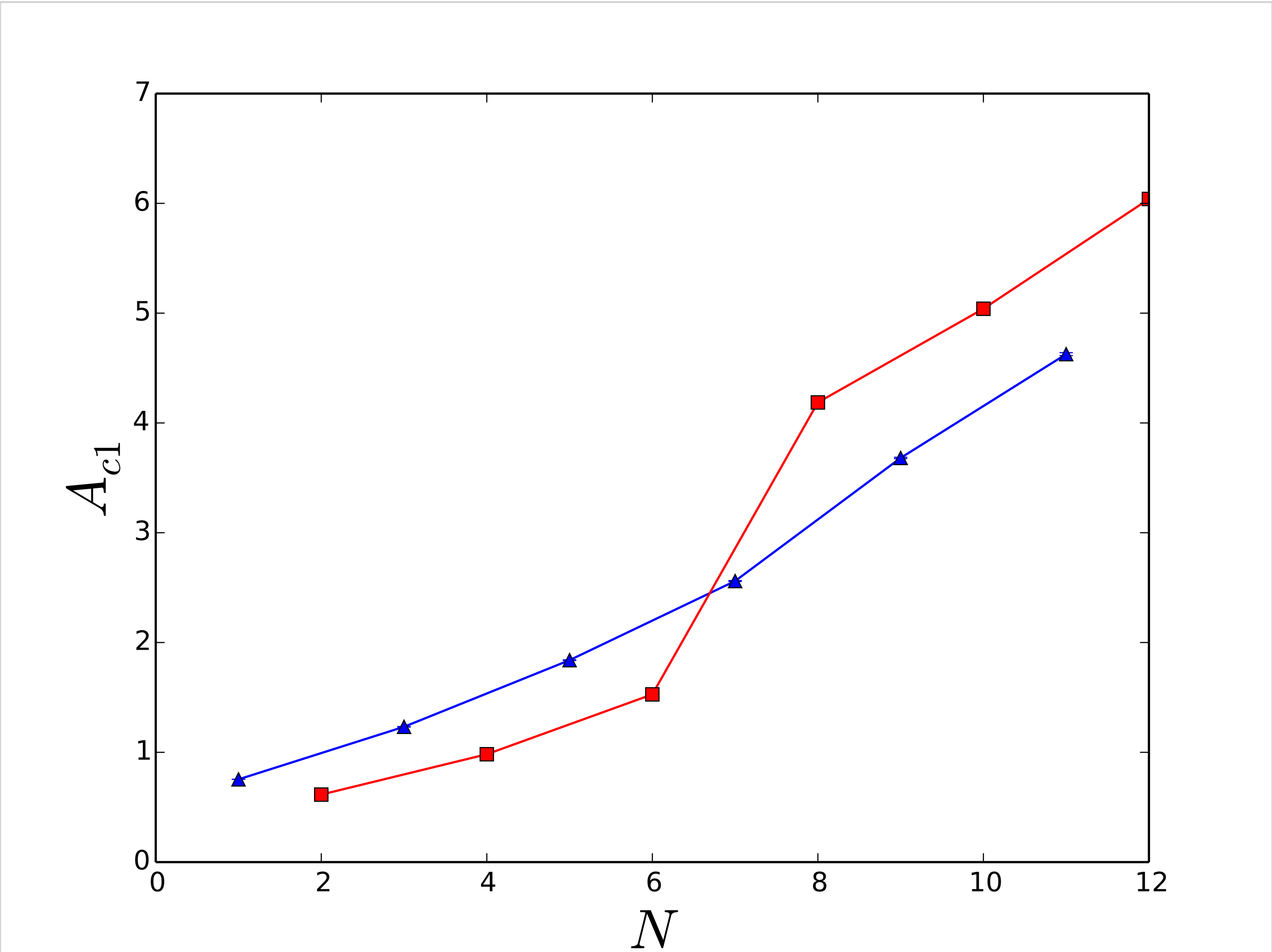
N=3

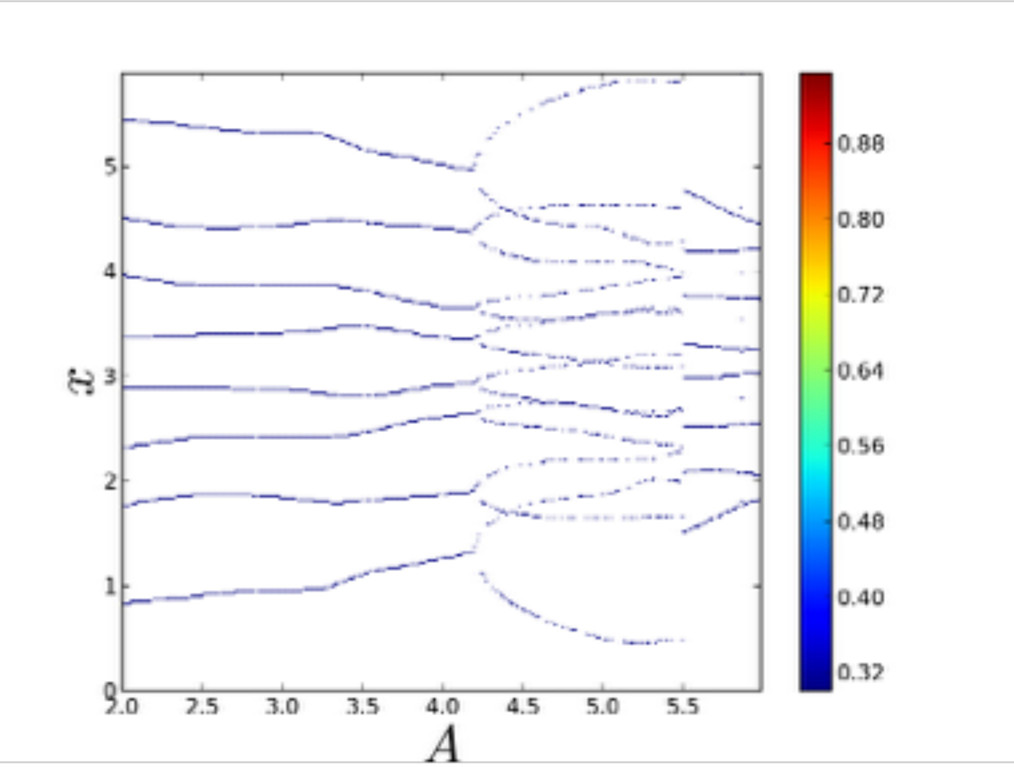
Antinode

For small *A* the time average force points in the direction of the antinodes of the potential

Type of first bifurcation changes from N=6 to N=7

N=8





Trying to understand transition

A different approach:

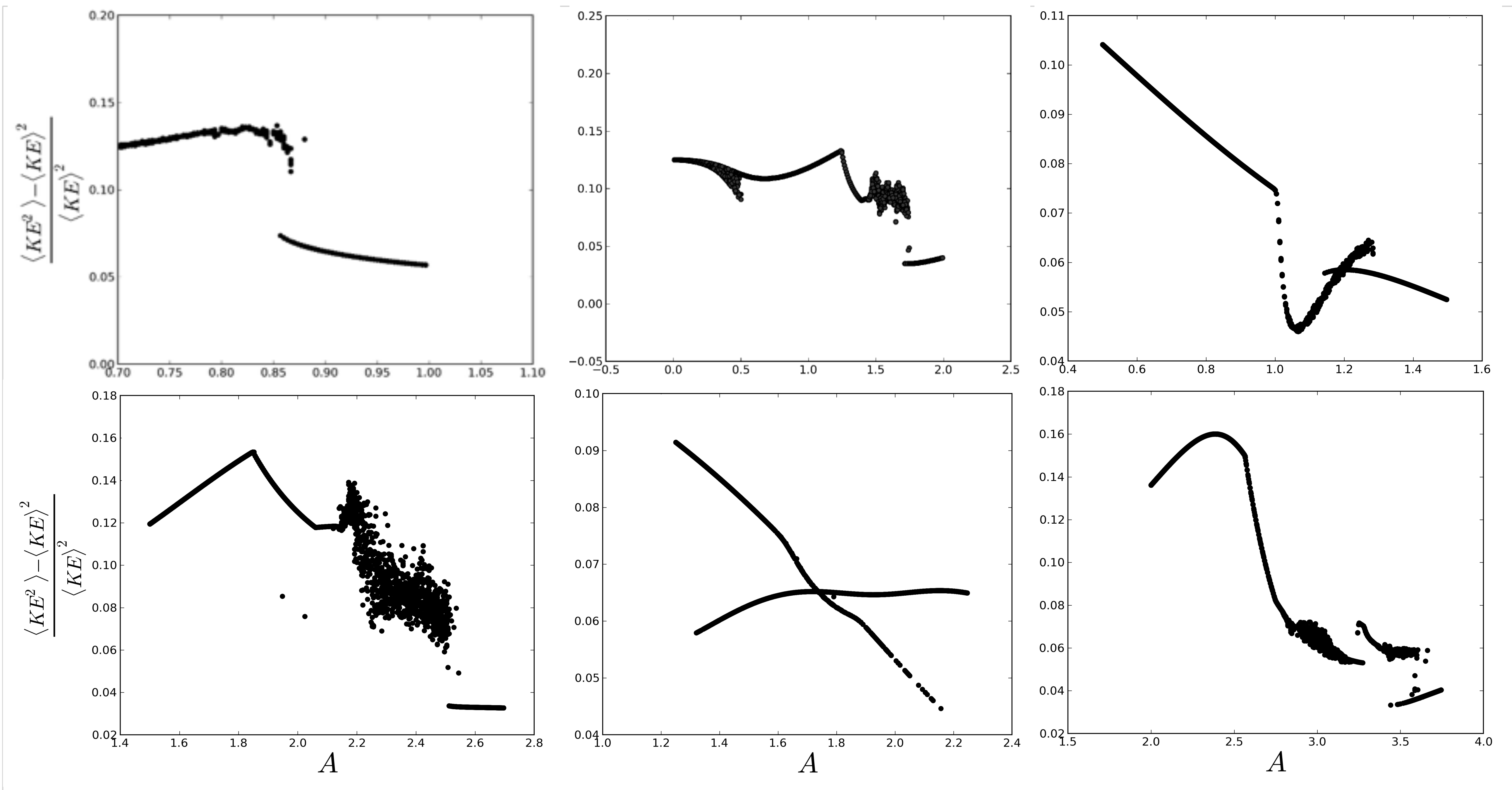
Assume nothing about the velocity distributions.

$$(\Delta KE)^2 = \frac{1}{4}\sum_{i,j}^{N}(\langle v_i^2 v_j^2\rangle - \langle v_i^2\rangle\langle v_j^2\rangle)$$

Squared Fractional Deviation

$$\frac{(\Delta KE)^2}{\langle KE\rangle^2}$$

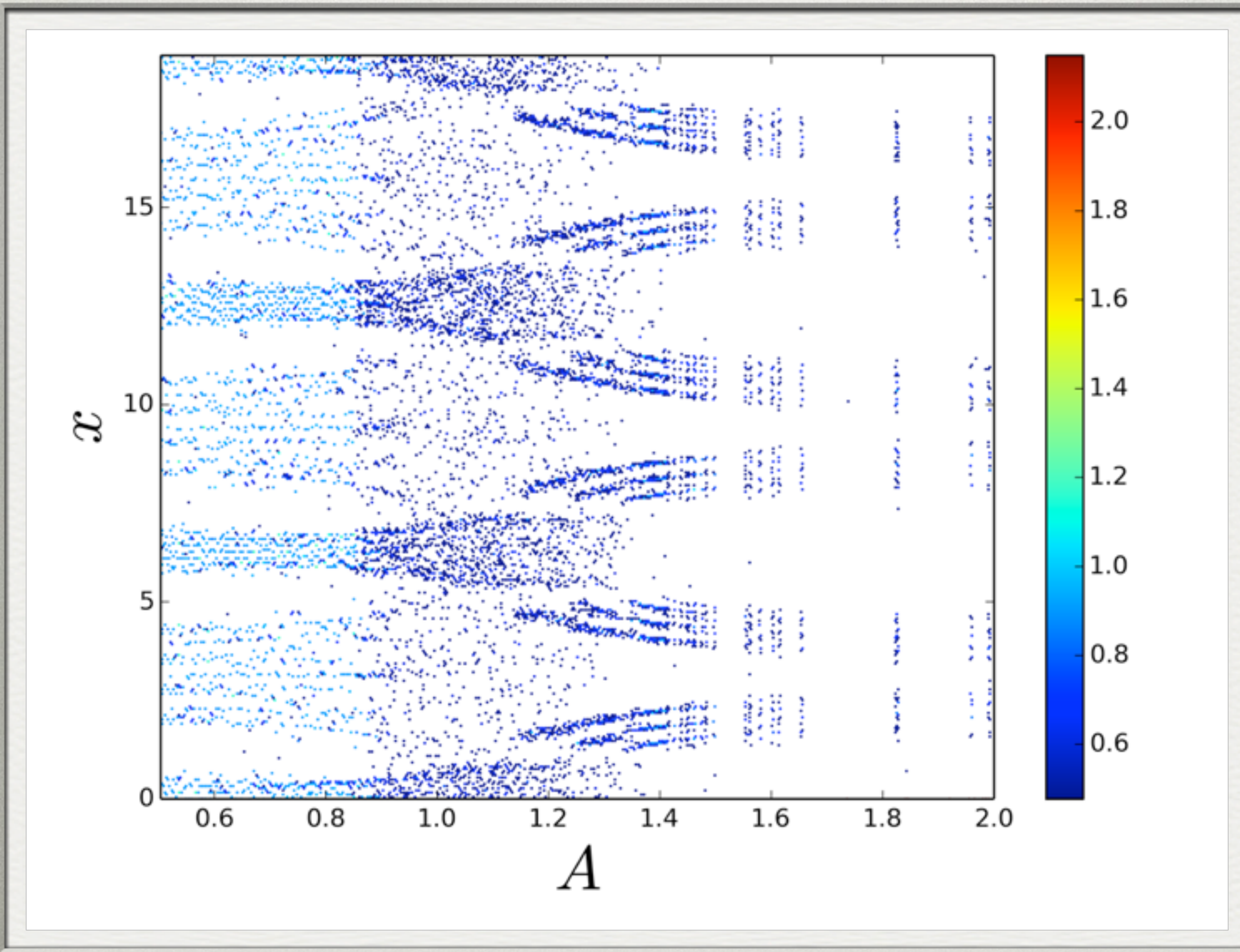
N=2 N=3 N=4

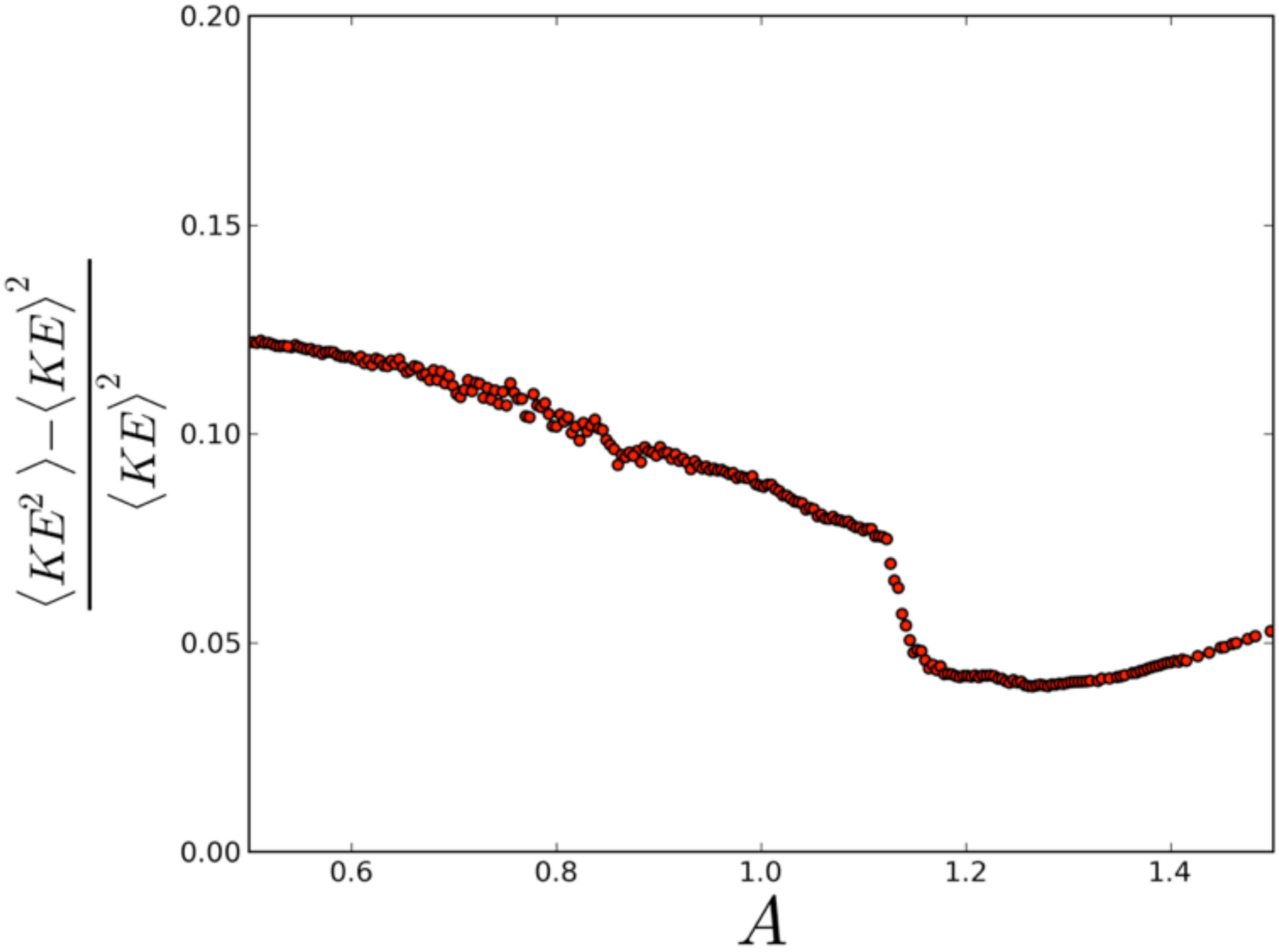


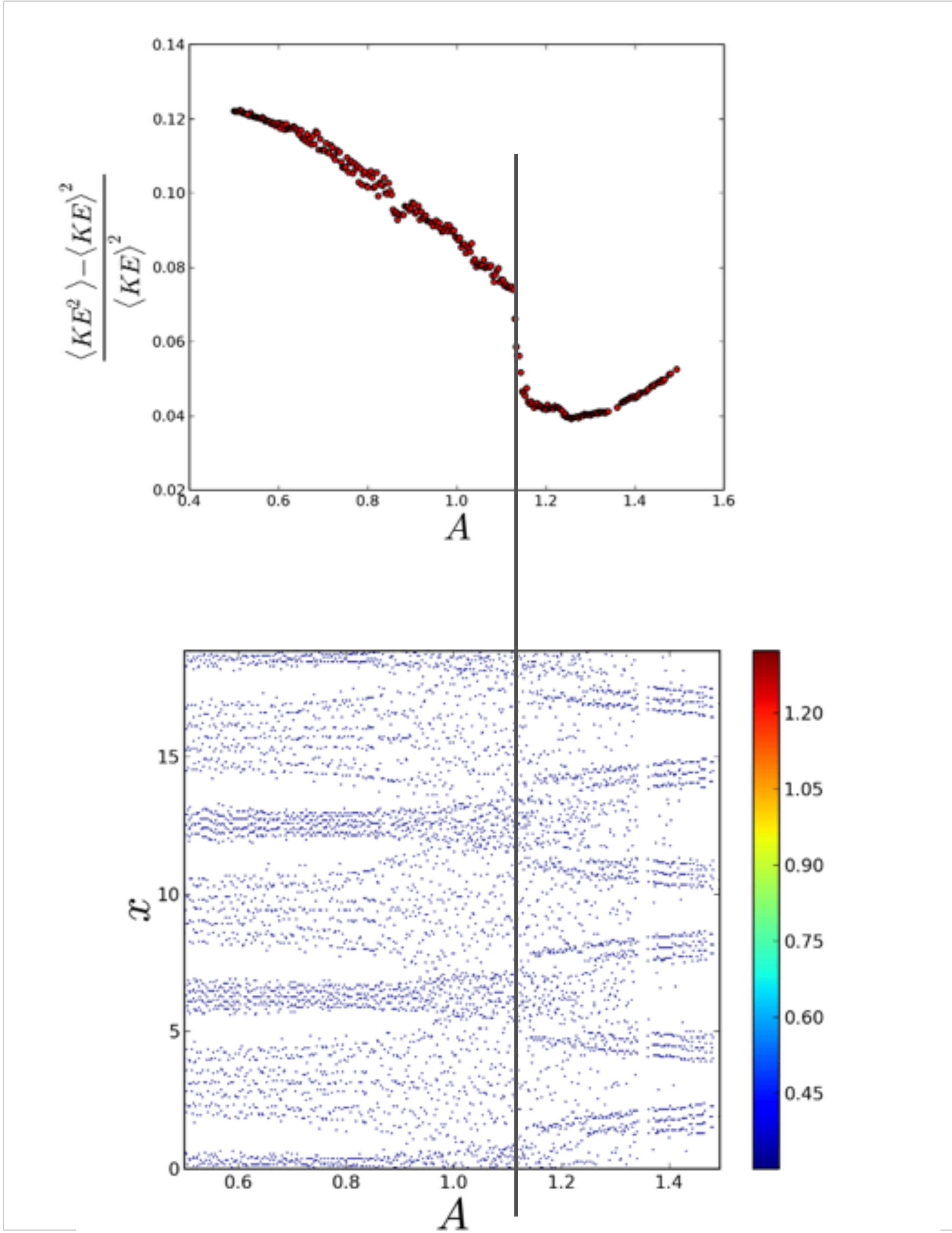
N=5 N=6 N=7

20 Particles in 3λ

$q^2 = .01$




0.20
0.15
0.10
0.05
0.00
0.6
0.8
1.0
1.2
1.4
A
$\frac{\langle KE^2 \rangle - \langle KE \rangle^2}{\langle KE \rangle^2}$



Clustering in “bifurcation” diagram matches the drop in effective number of degrees of freedom

Outline

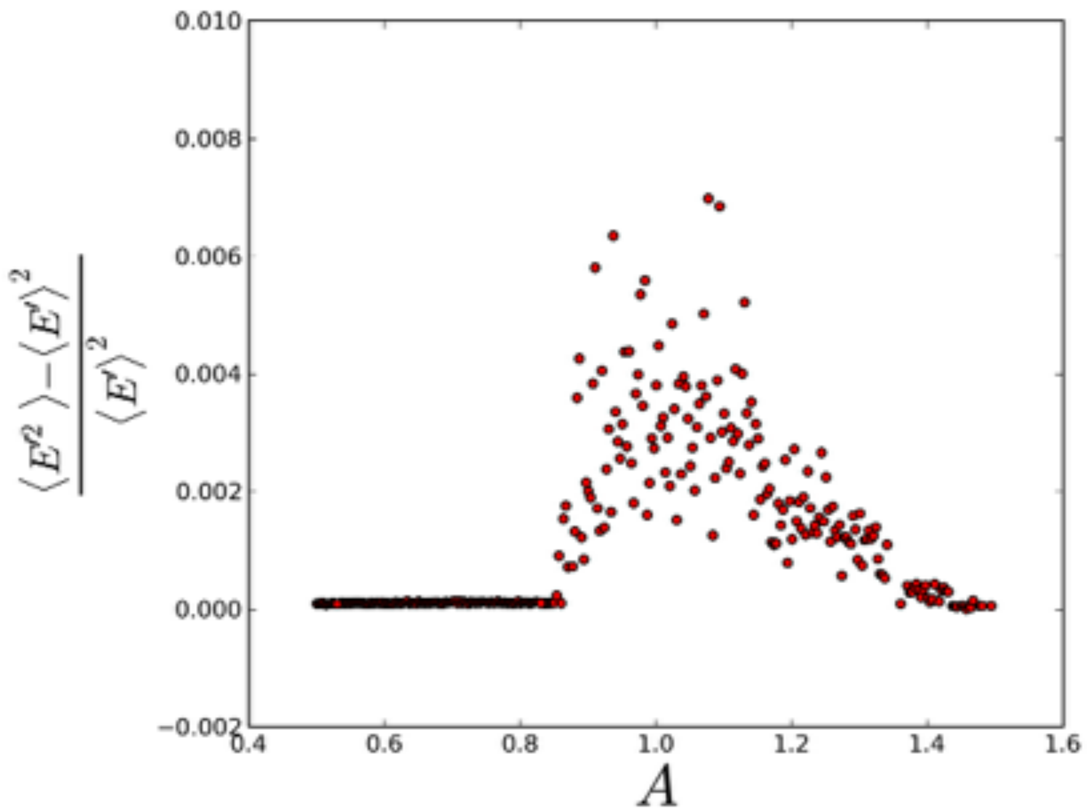
1) The electric curtain: Experimental, Numerical.

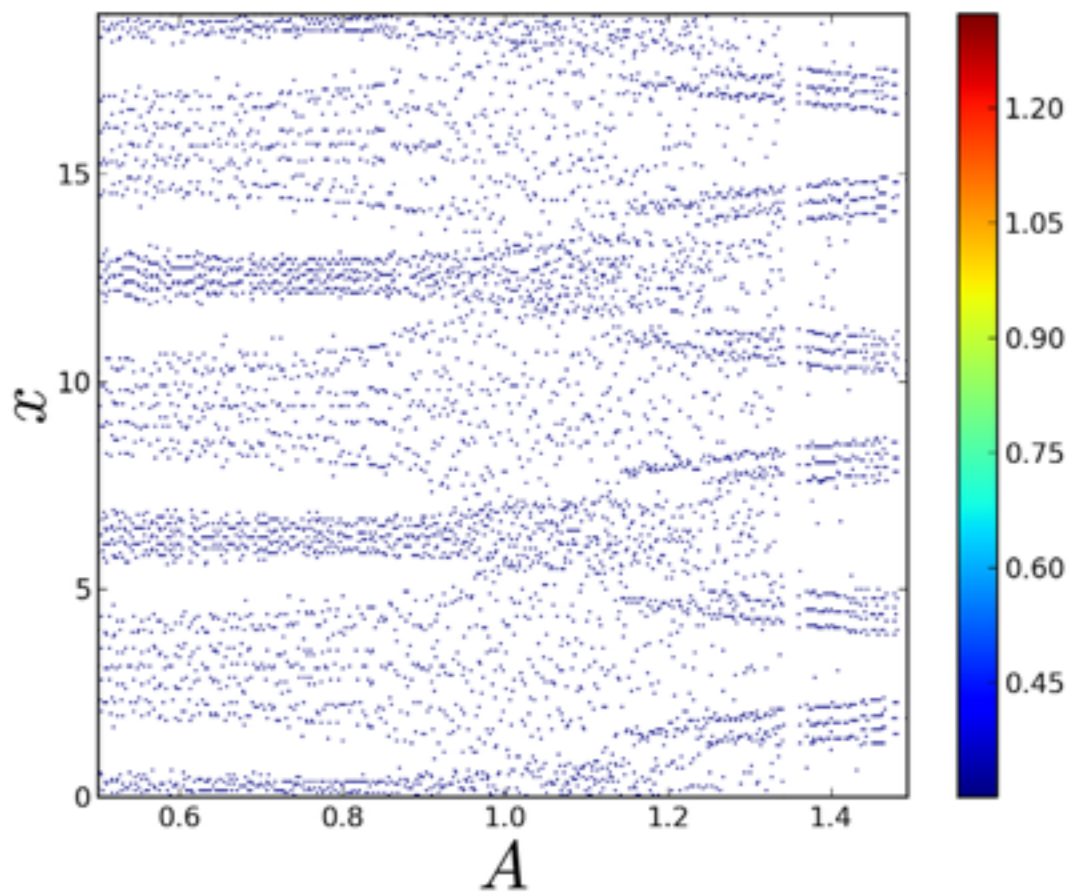
2) What are STP potentials?

3) Current methods of understanding.

4) The simple model: Work on the fundamentals.

5) Proposal.

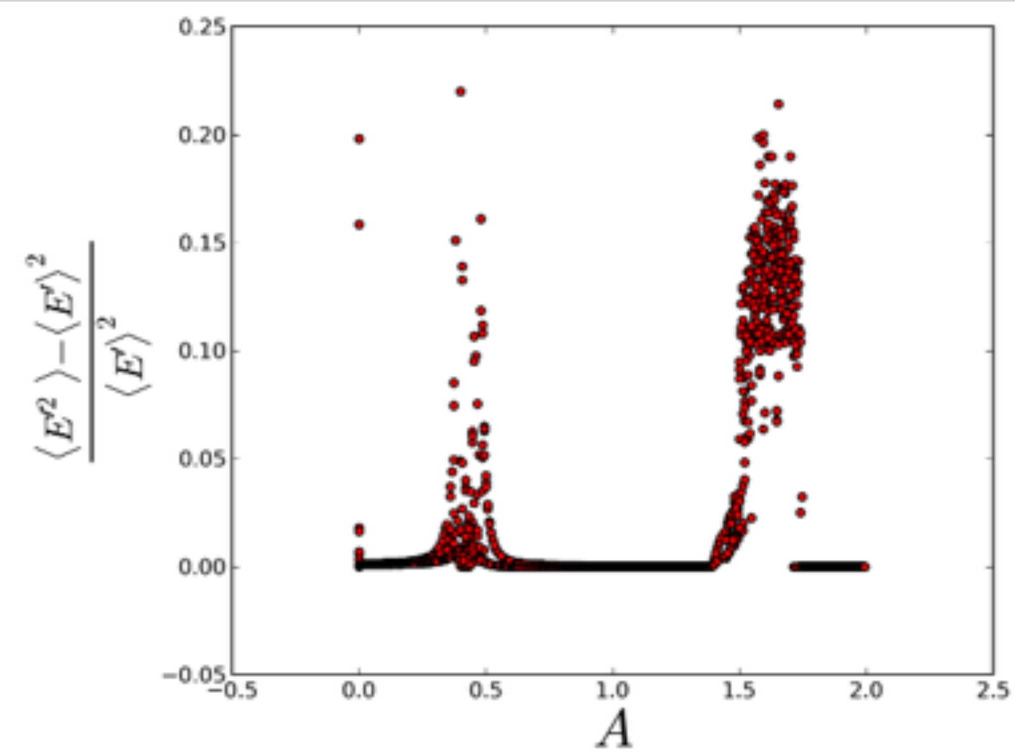




E′ **is the kinetic energy in the Poincaré sections** **<u>only</u>**

Another example:

N=3



We want to understand the particle statistics in the Poincaré sections

- Need to compare velocity correlations in the Poincaré sections to those in the full system.
- We need an analytical understanding of the results.

Luca D'Alessio, Anatoli Polkovnikov, "Many-body energy localization transitions in periodically driven systems", Annals of Physics, **333** (2013)

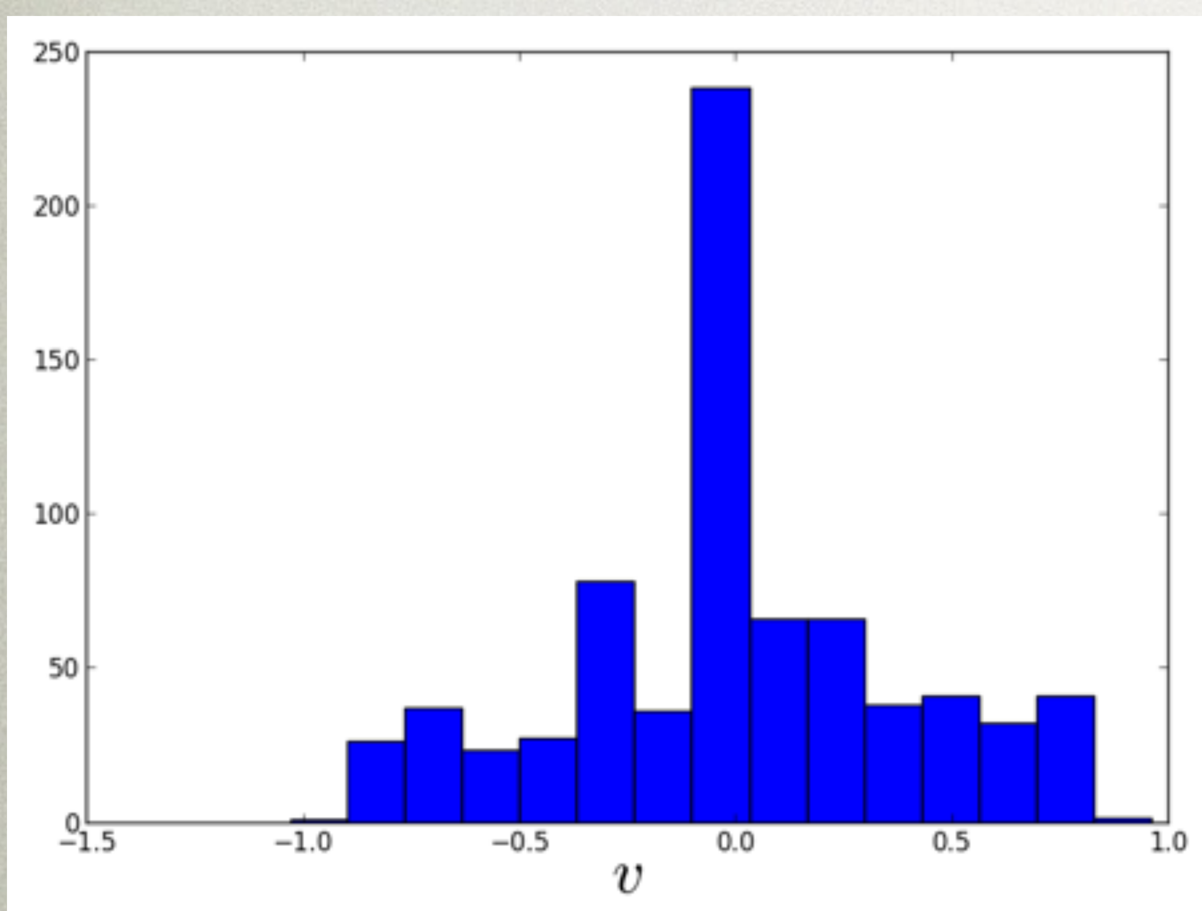
Acknowledgments



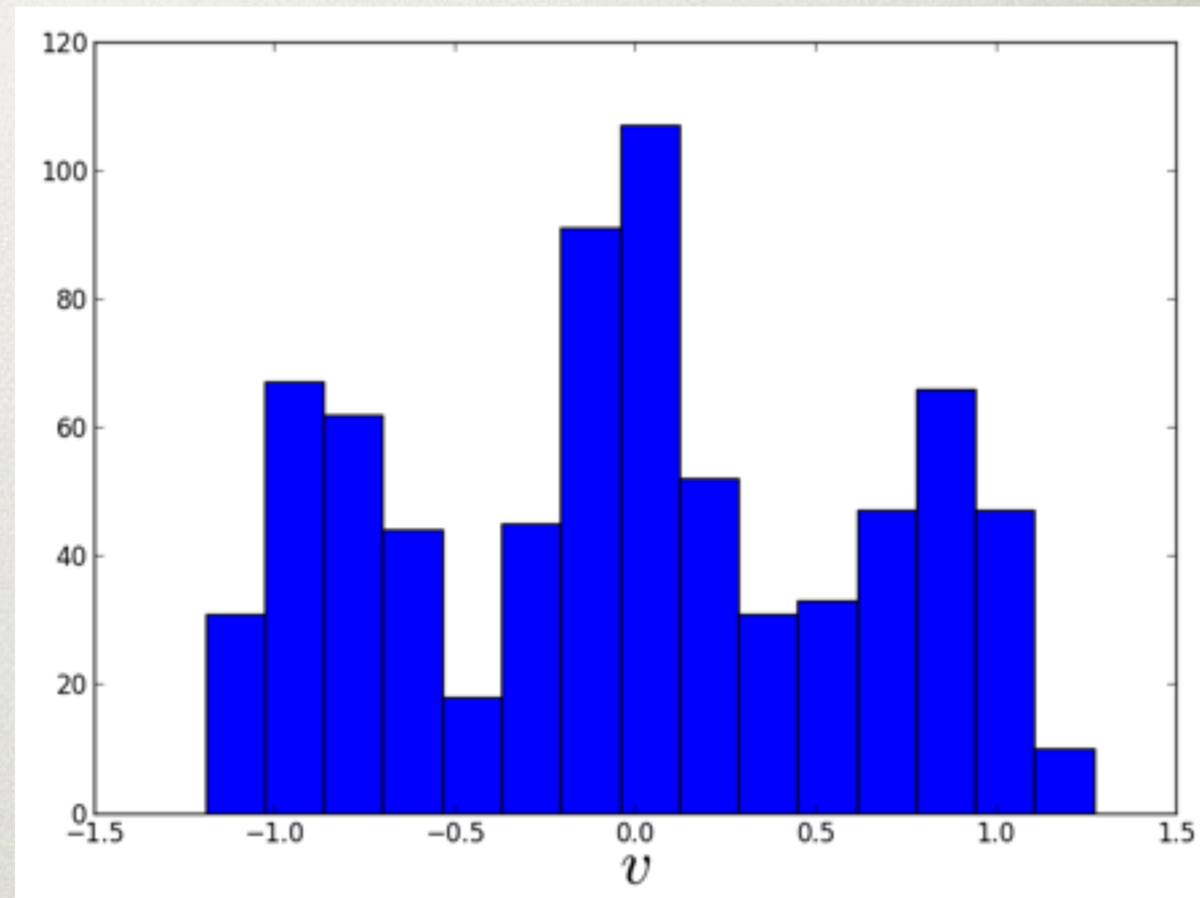

UVM
Physics
Department

N=5

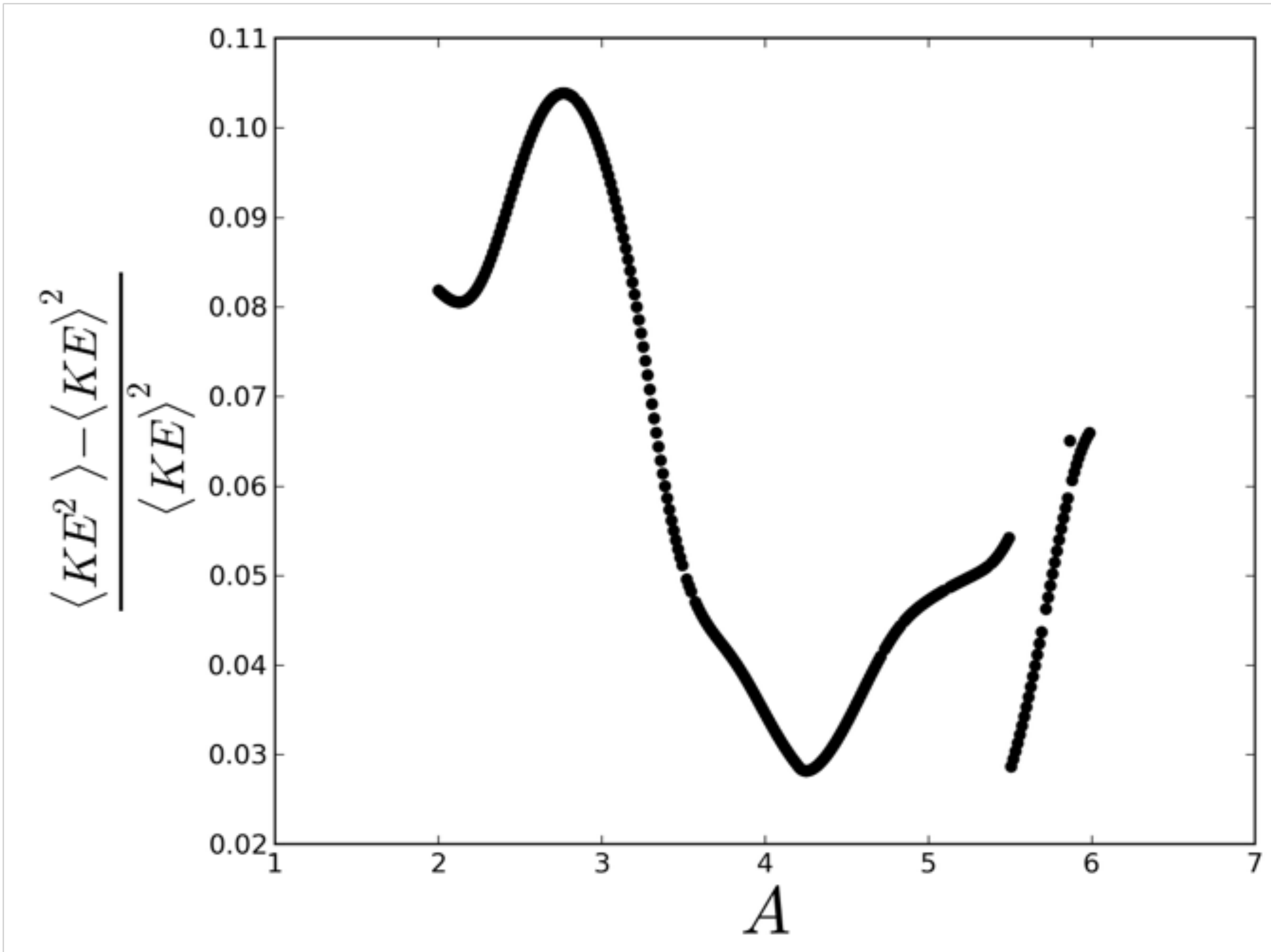
Before Transition



After Transition

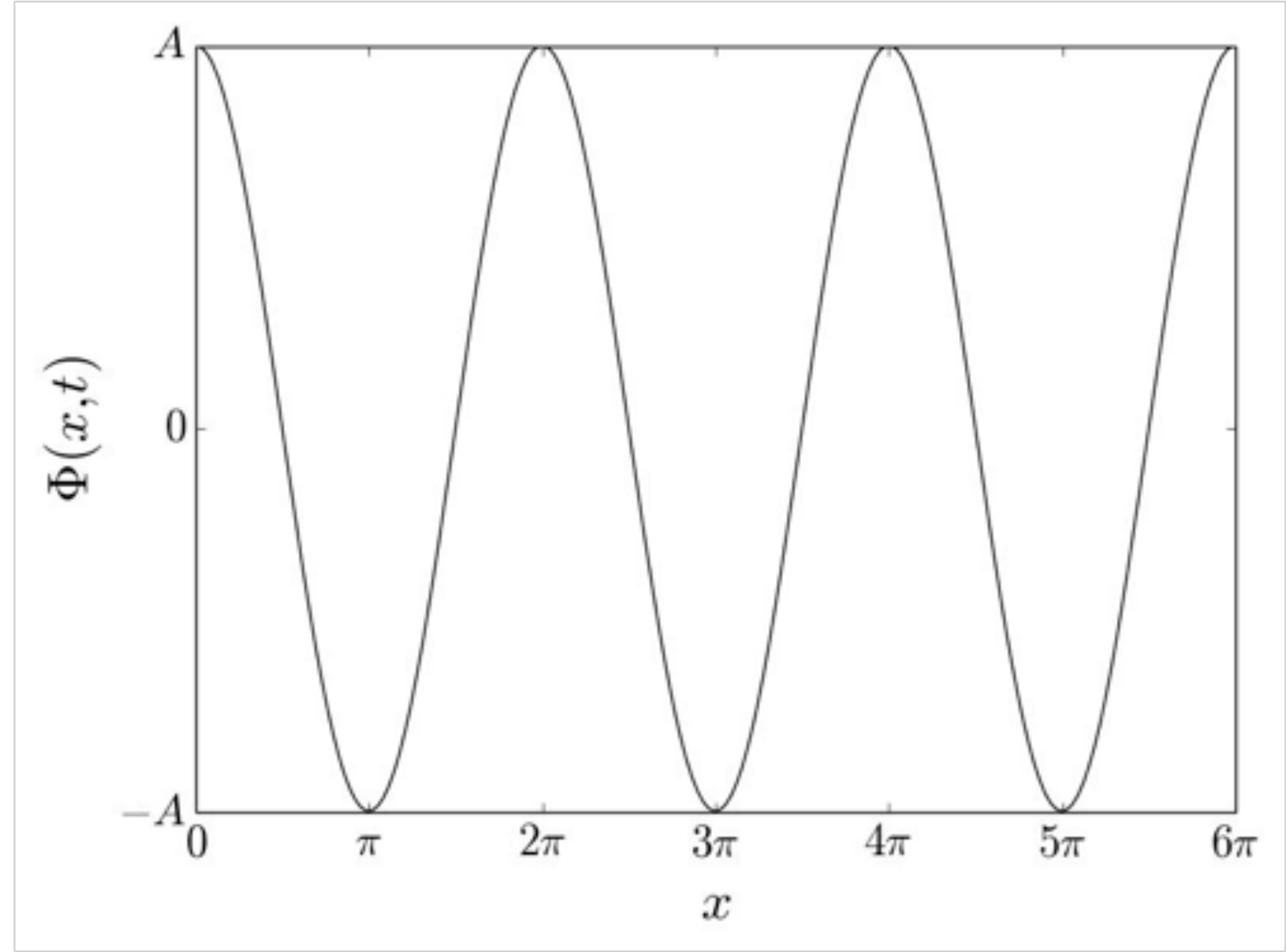


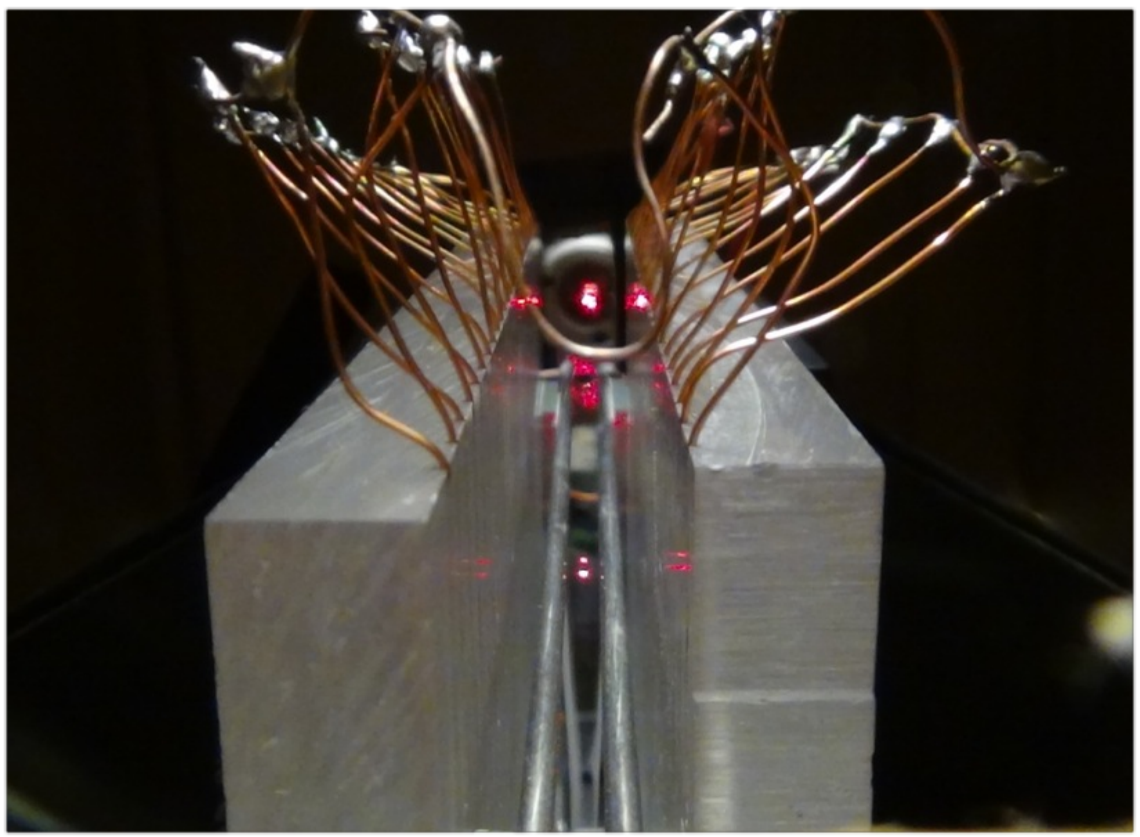
N=8



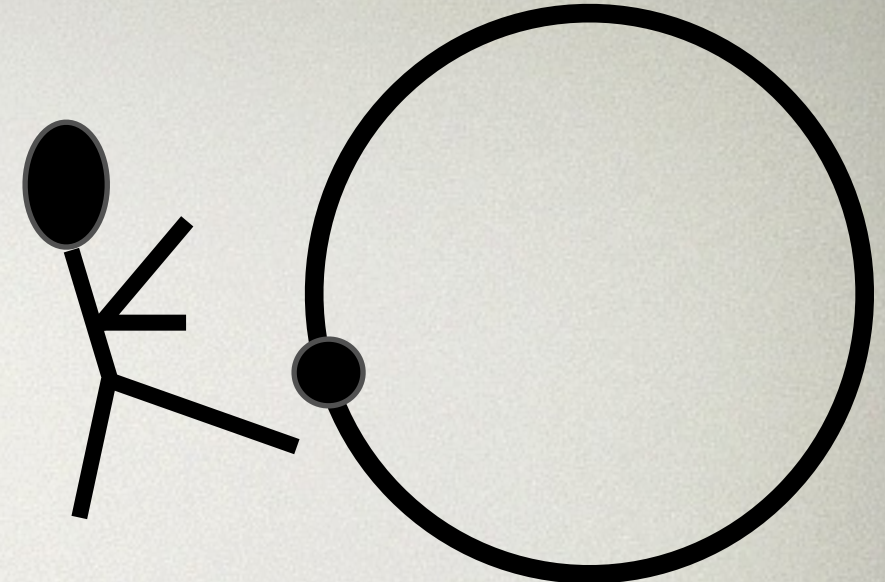
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
1
2
3
4
5
6
7
A
$\frac{\langle KE^2 \rangle - \langle KE \rangle^2}{\langle KE \rangle^2}$

What does this potential look like?



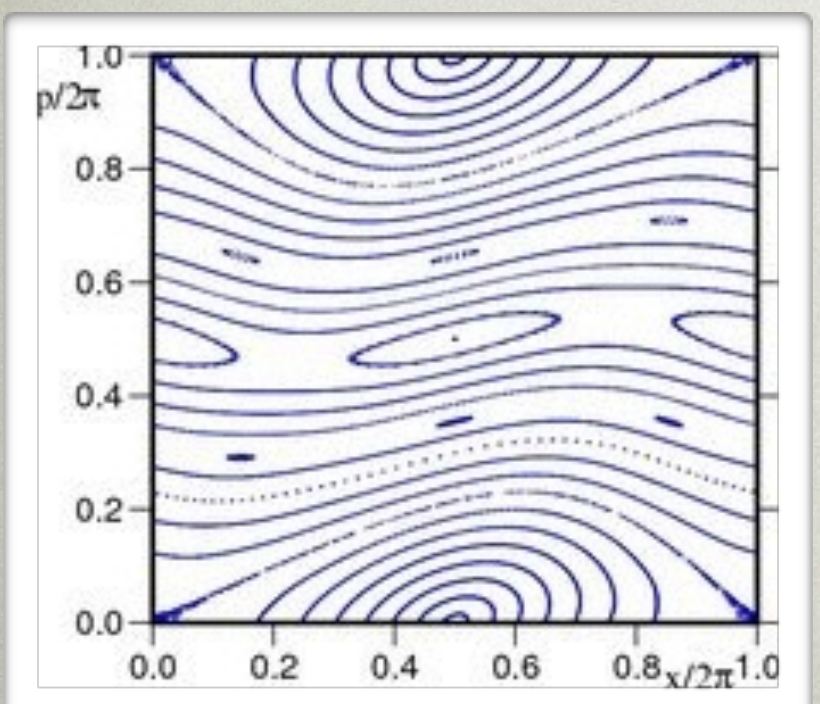


The kicked rotator described by

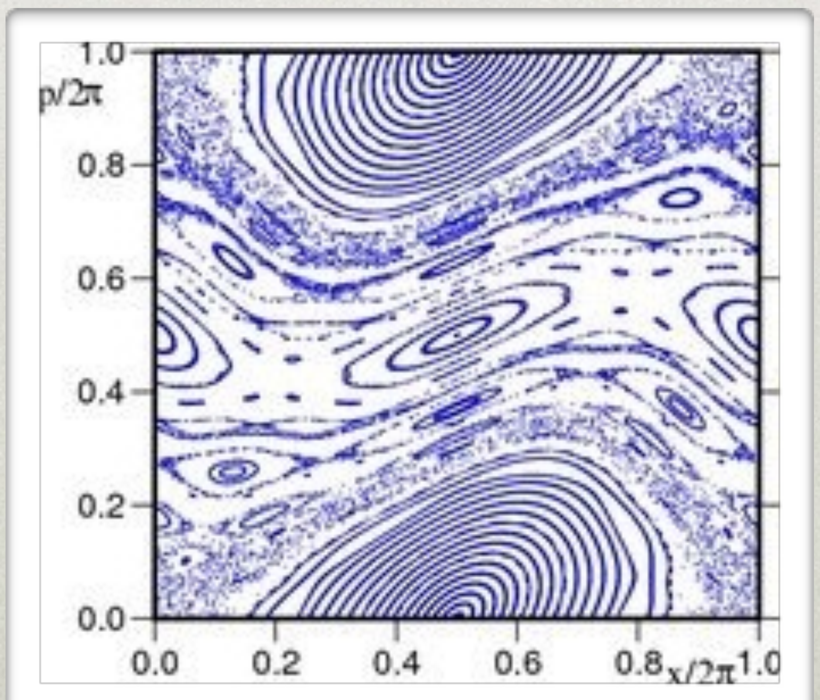


$$H = p^2/2I + k\cos\theta \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

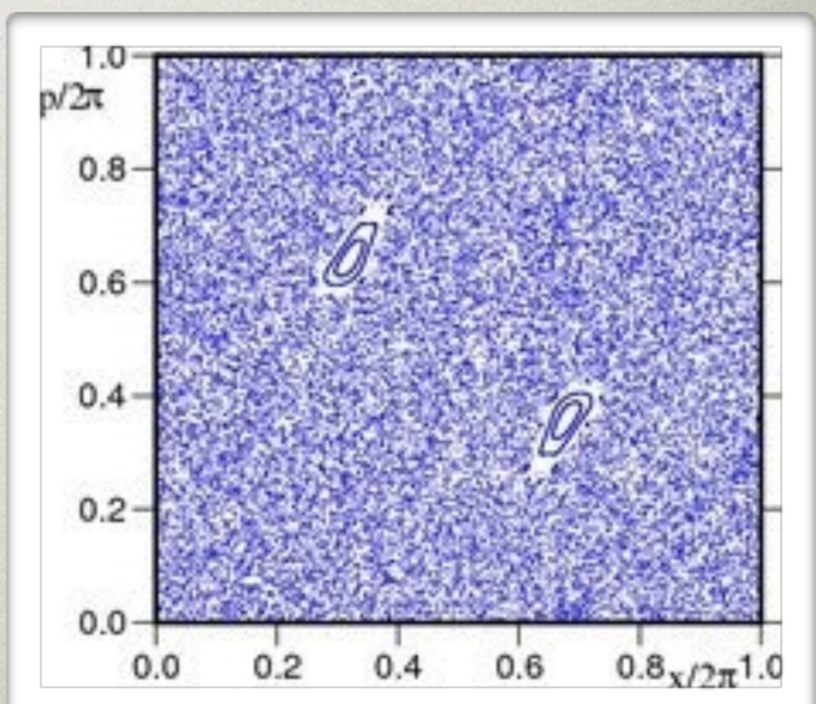
Images from scholarpedia



k=0.5

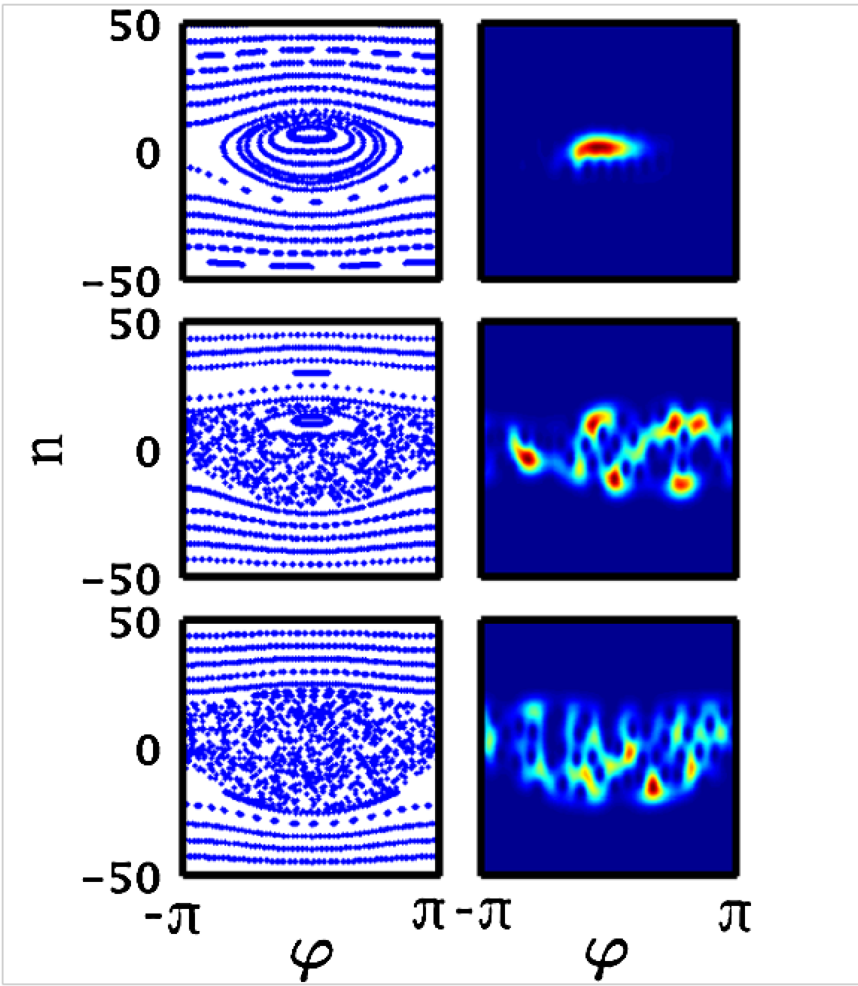


k=0.972

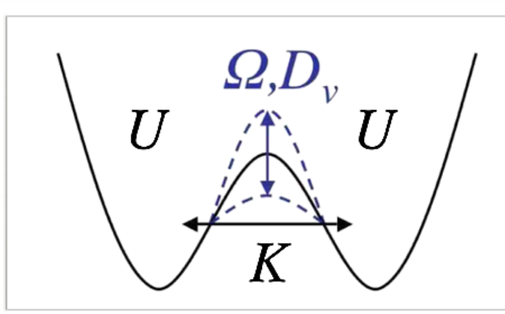


k=5.0

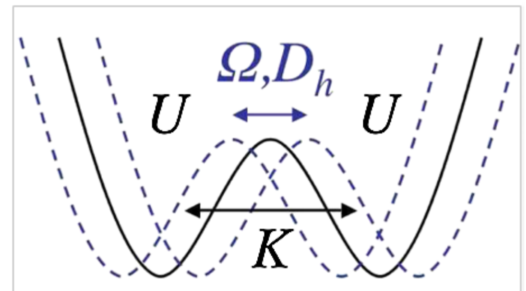
E. Boukobza, M. G. Moore, D. Cohen, A. Vardi (PRL, 2010)

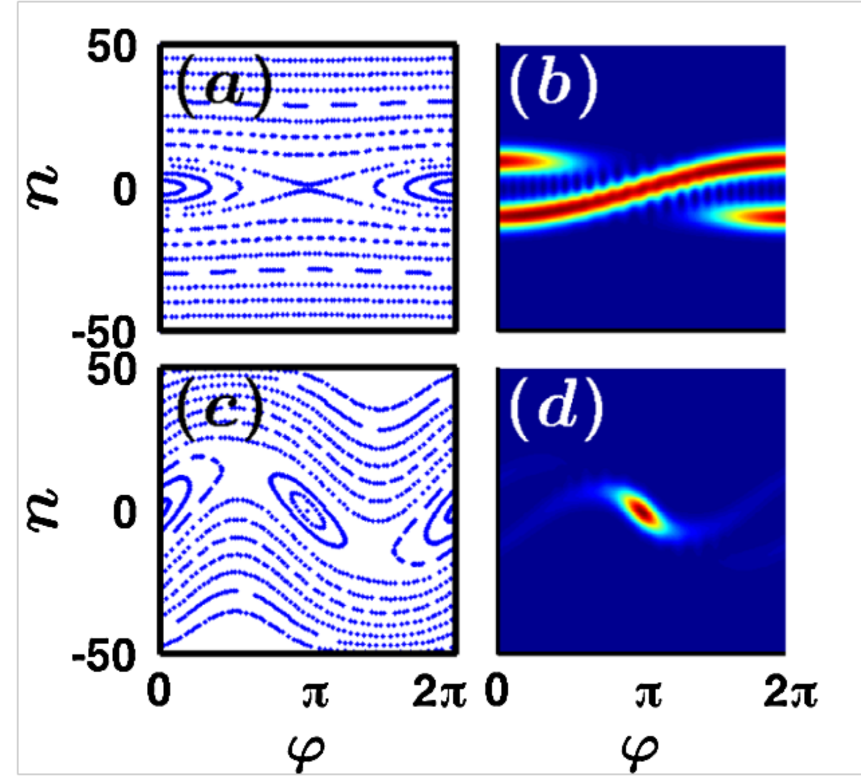


Horizontal driving



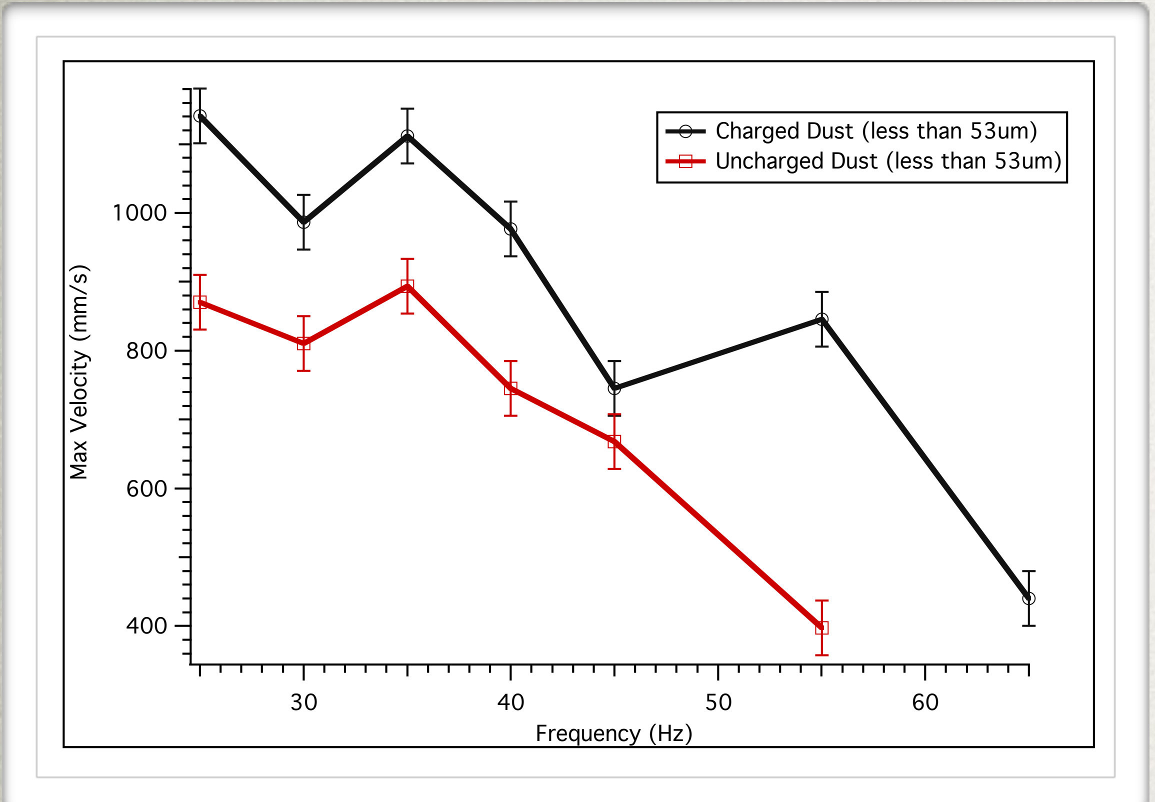
Vertical driving





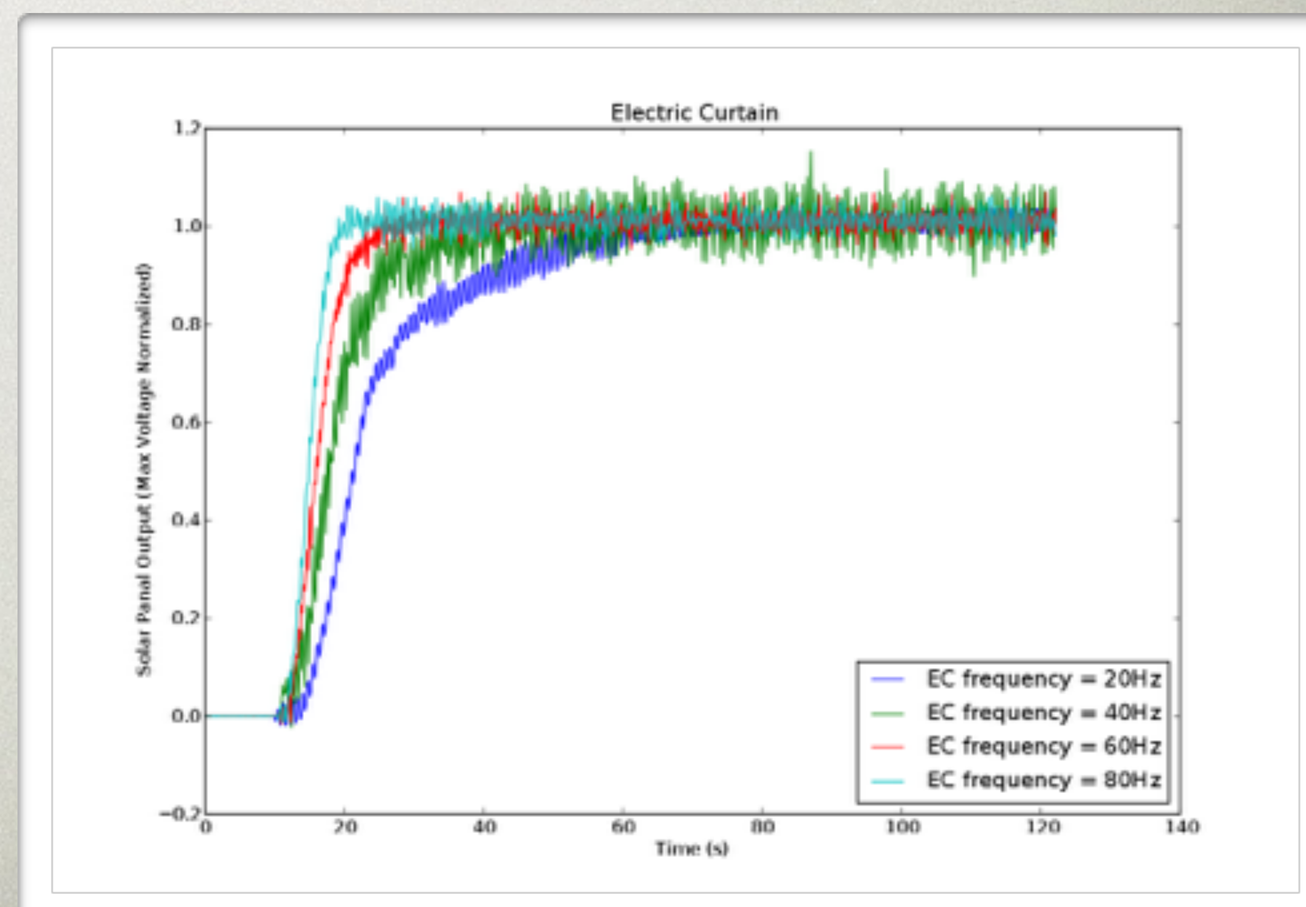
Applications in atom interferometers that could potentially resolve phase shifts below the standard quantum limit (1/√N). They would be limited by the Heisenberg fundamental limit (1/N).

Velocity Measurements



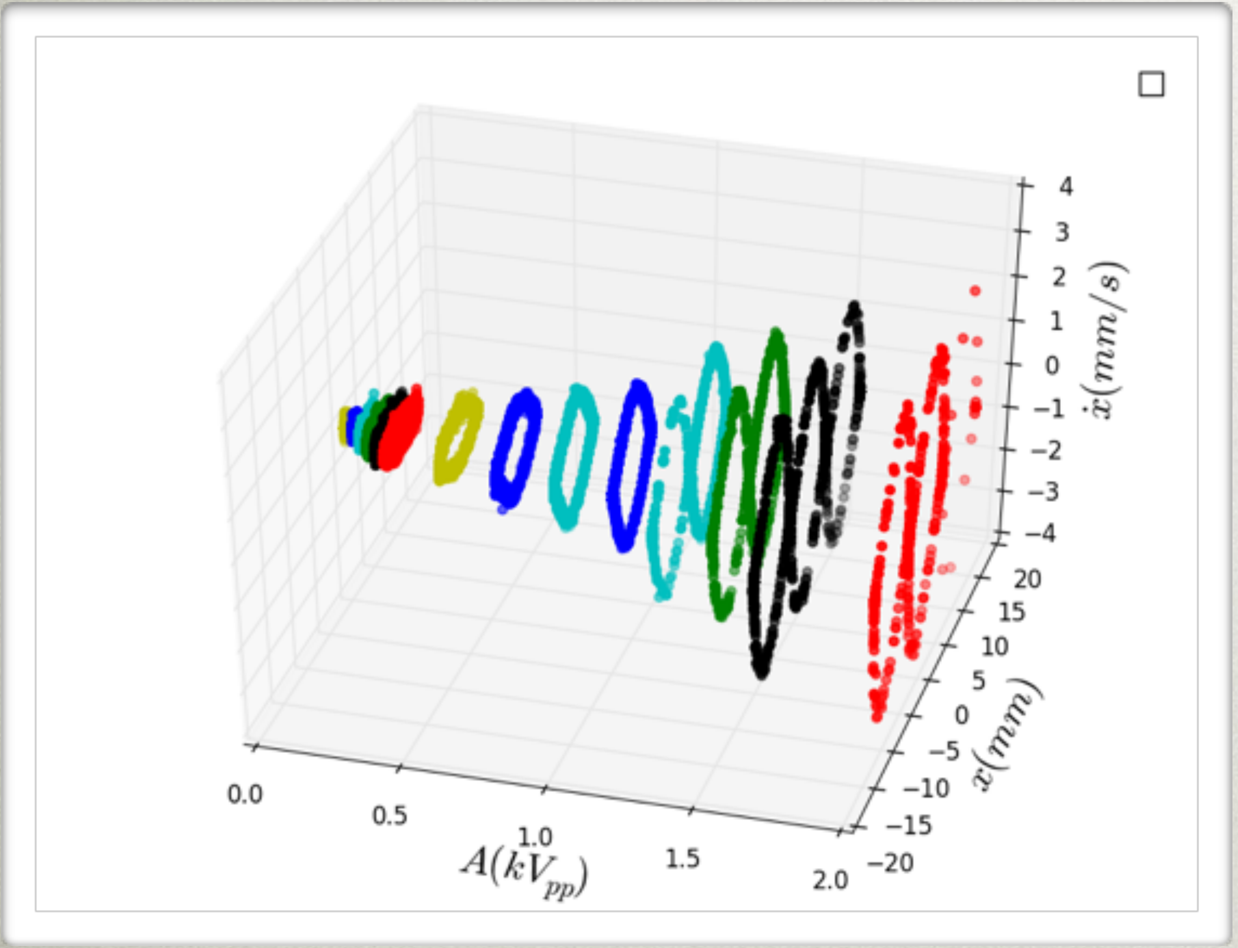
Include low frequency vibration?

Solar panal applications

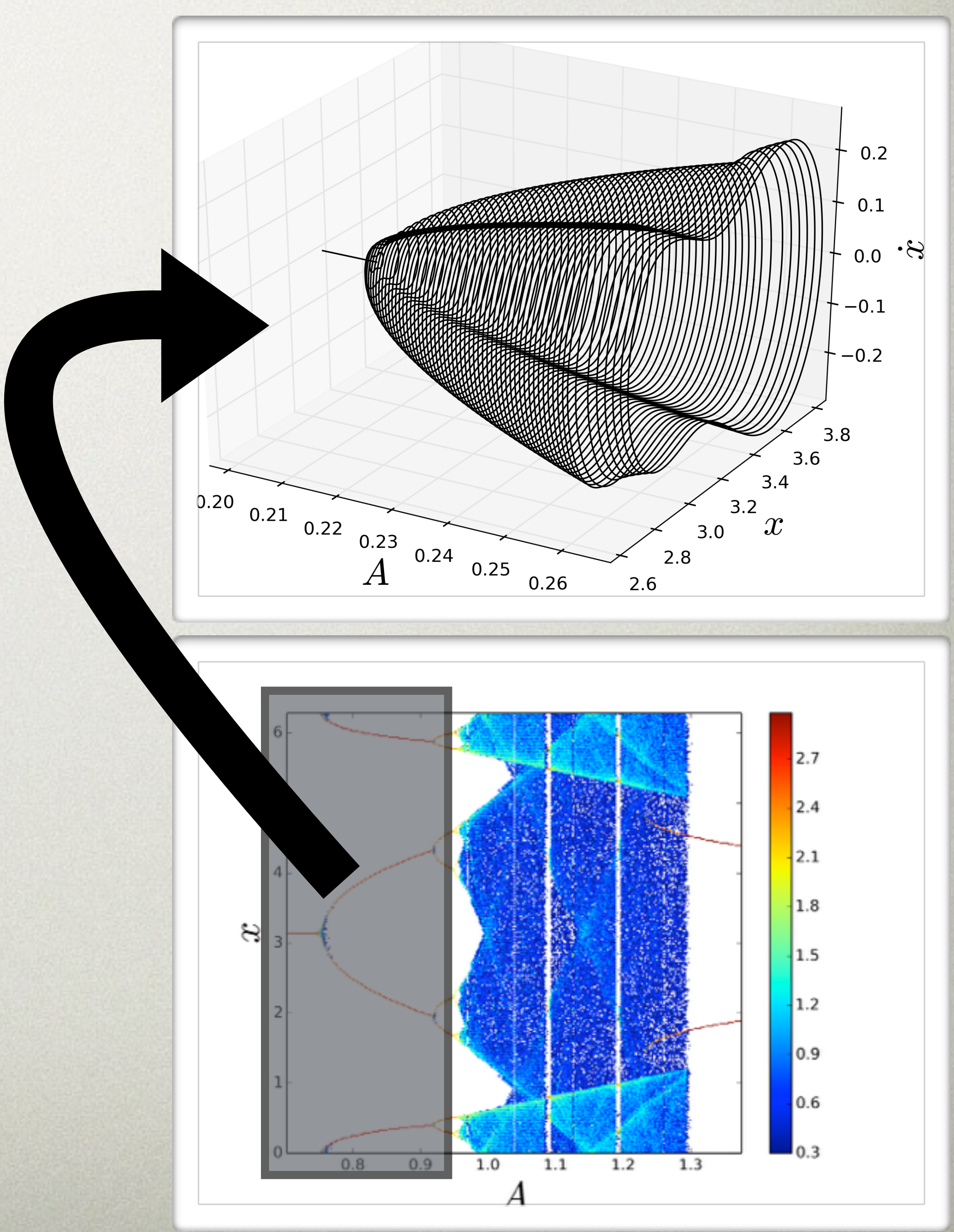


First Single Particle Bifurcations

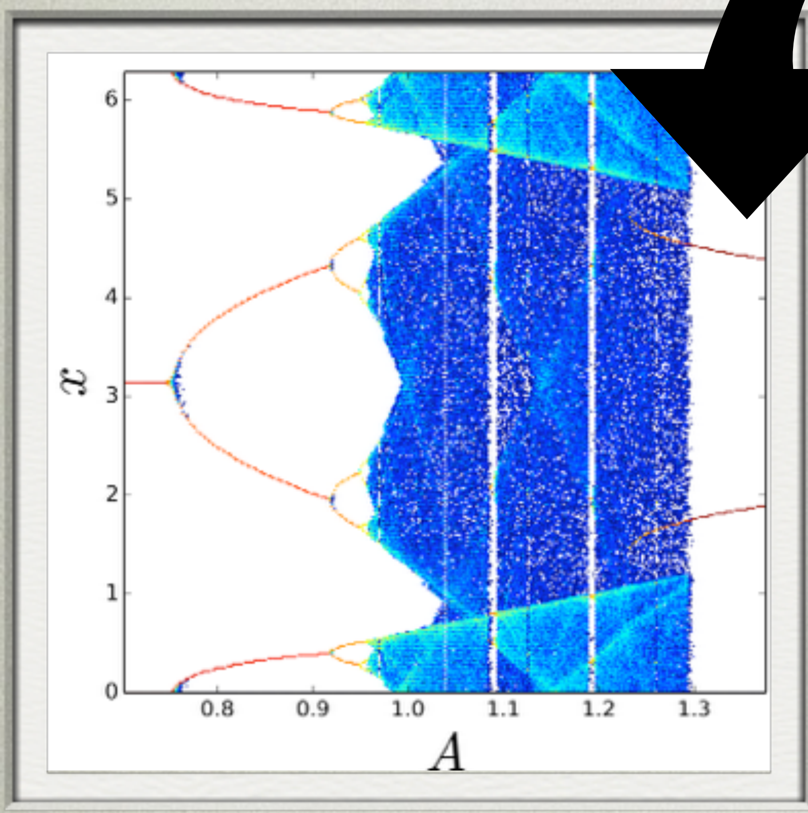
Experimental

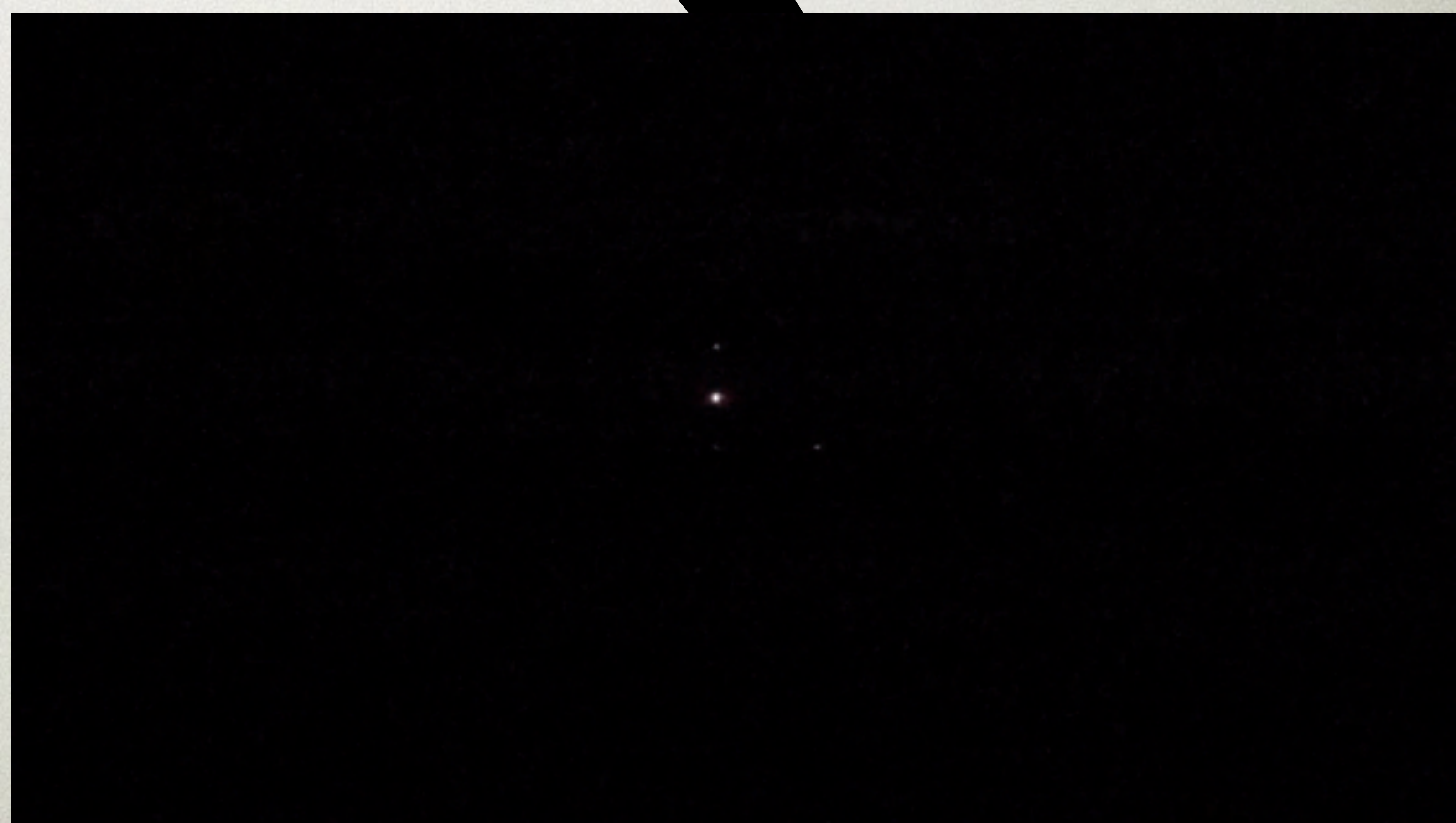


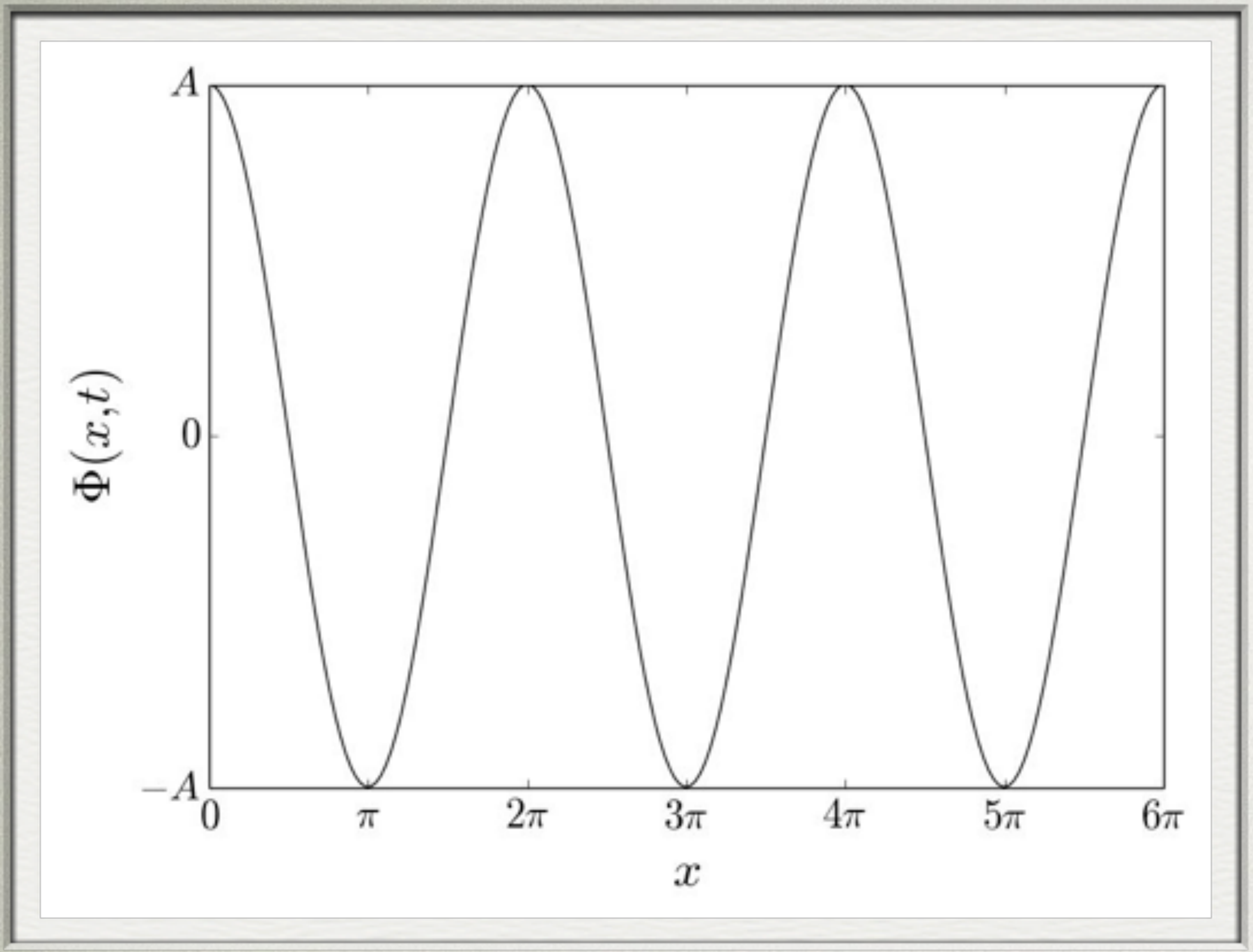
Numerical

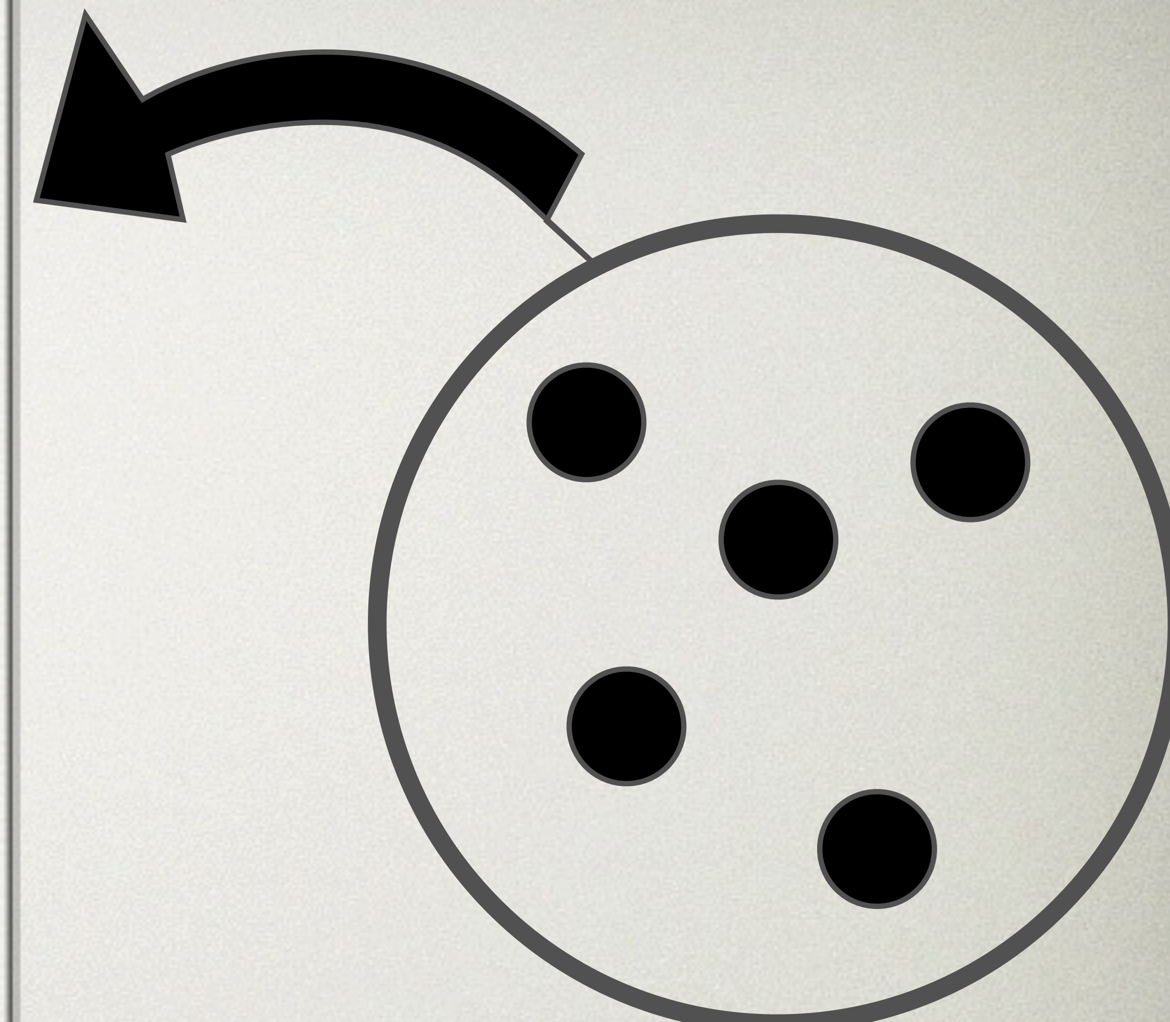


Propagating Trajectory









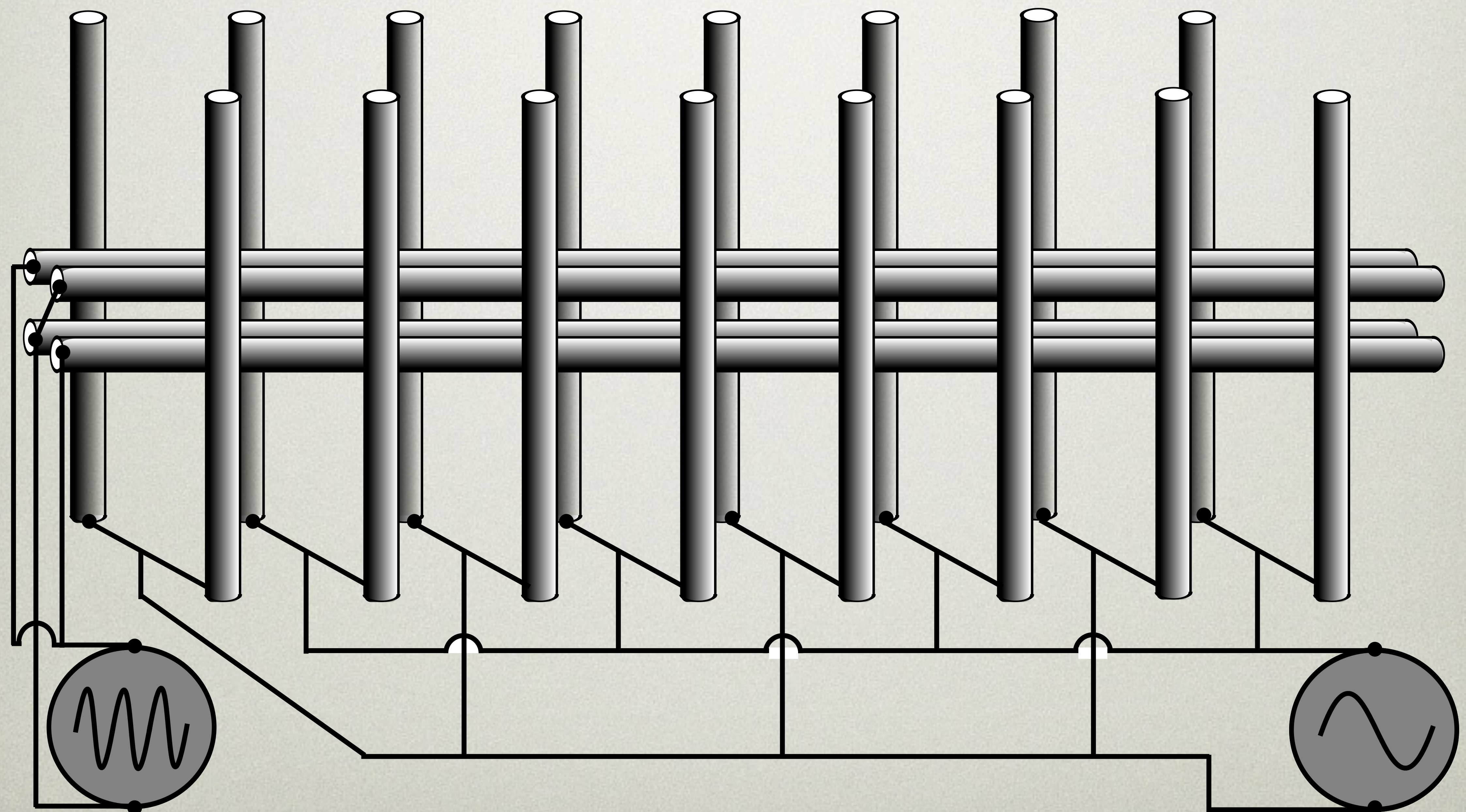
Equations of motions

$$\ddot{\vec{x}}_i = \frac{1}{2}\sum_{j\neq i}^{N}\frac{q^2\hat{r}}{r_{i,j}^2} - \beta\dot{\vec{x}} + \nabla\Phi$$

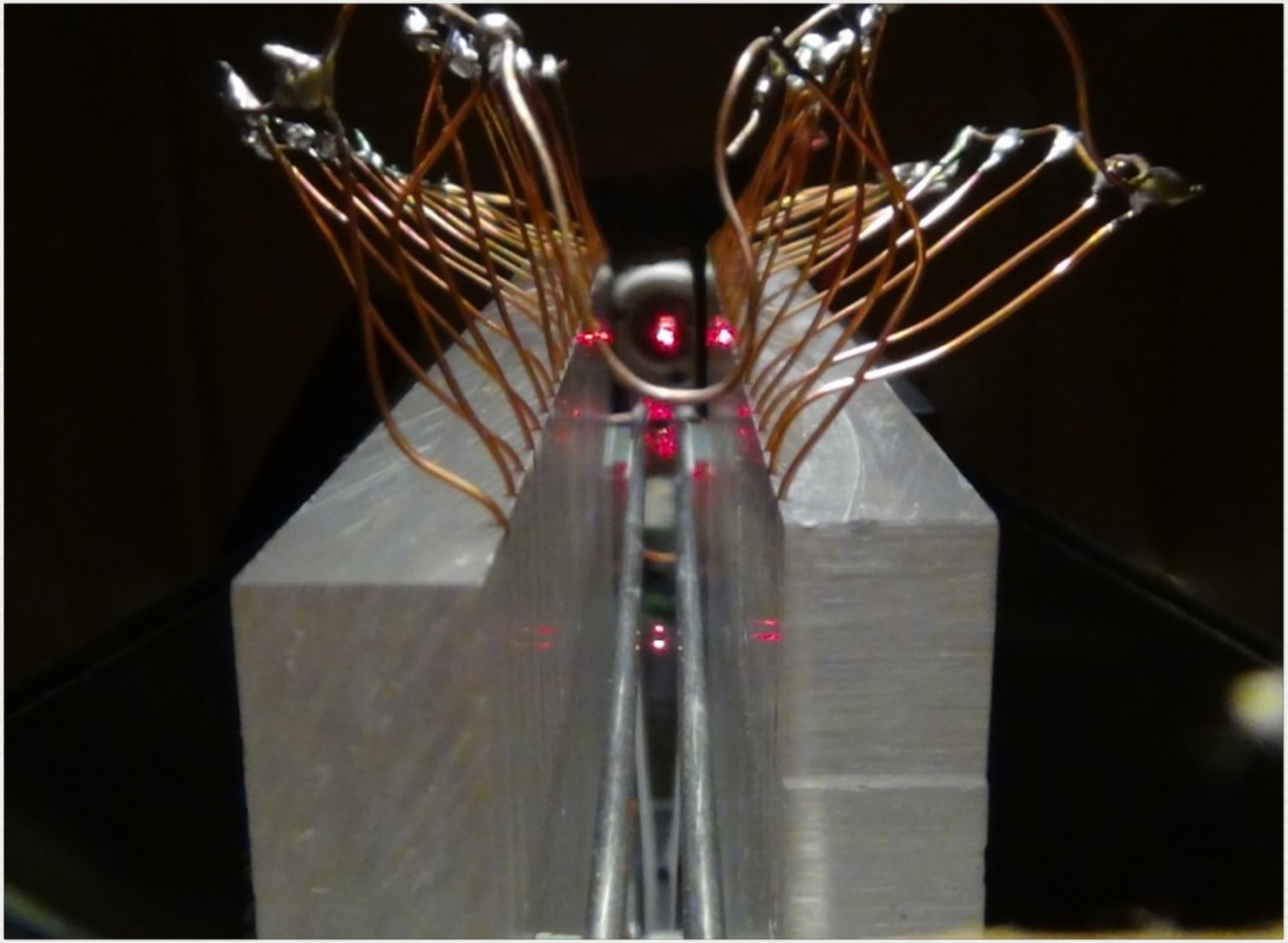
Importance of Understanding STP Potentials

- Transport control & ratchets
    - Hamiltonian, H. Schanz, M. F. Otto, R. Ketzmerick, T. Dittrich (PRL, 2001)
    - Damped, Jose L. Mateos (PRL, 2000)
- Josephson Junctions,
  E. Boukobza, M. G. Moore, D. Cohen, A. Vardi (PRL, 2010)
- Dynamic stabilization and potential renormalization,
  A. Wickenbrock, P. C. Holz, N. A. Abdul Wahab, P. Phoonthong, D. Cubero, F. Renzoni(PRL, 2012)
- Dust mitigation and control in extraterrestrial environments (NASA),
  O. Myers, J. Wu, J. Marshall (JAP, 2013).

Experimental Setup

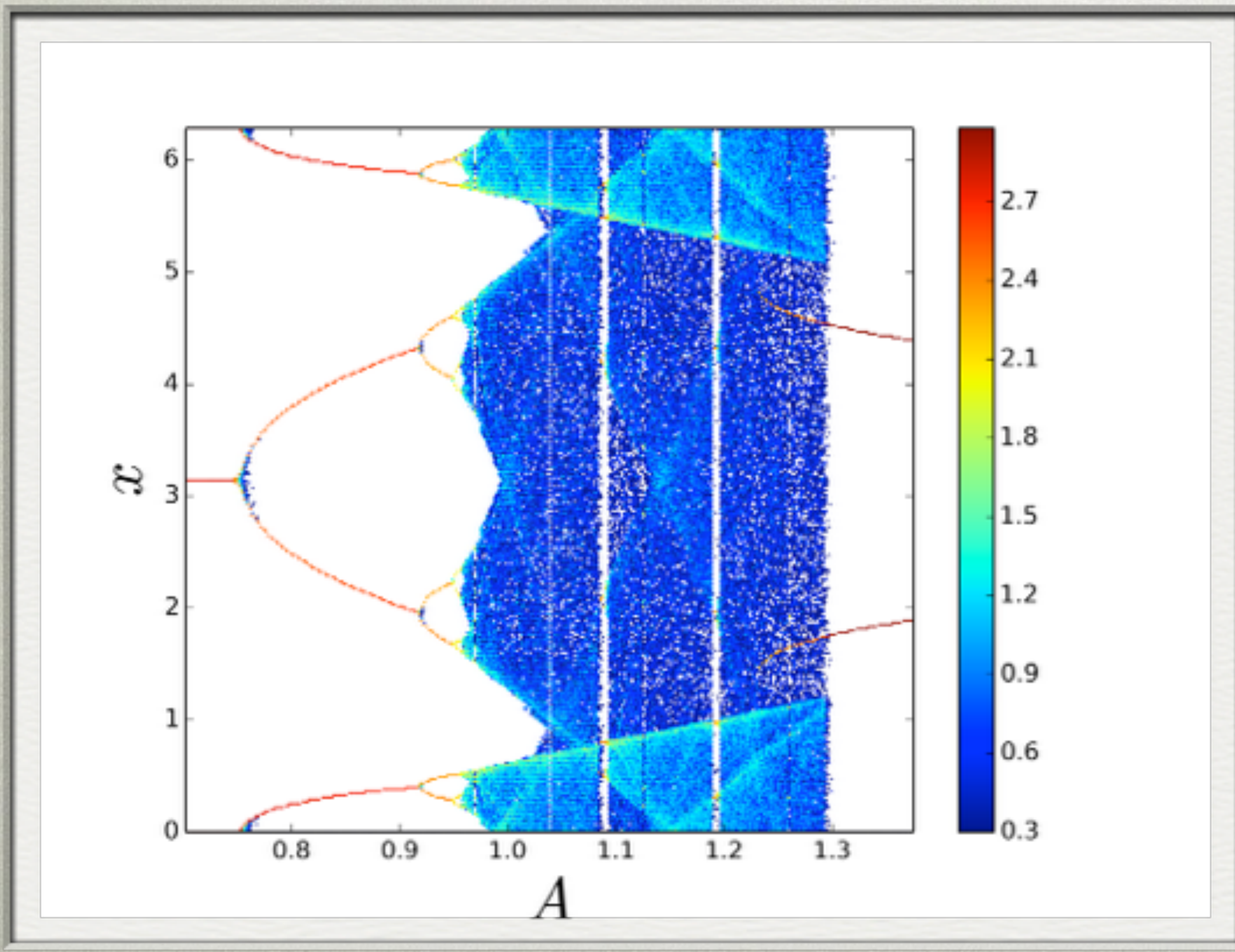


Two Particles in Apparatus



Model Spatiotemporally Periodic Potential

$$\Phi(x,t) = -A\cos x \cos t$$



$\lambda = 2\,\pi$

concentration = N/n

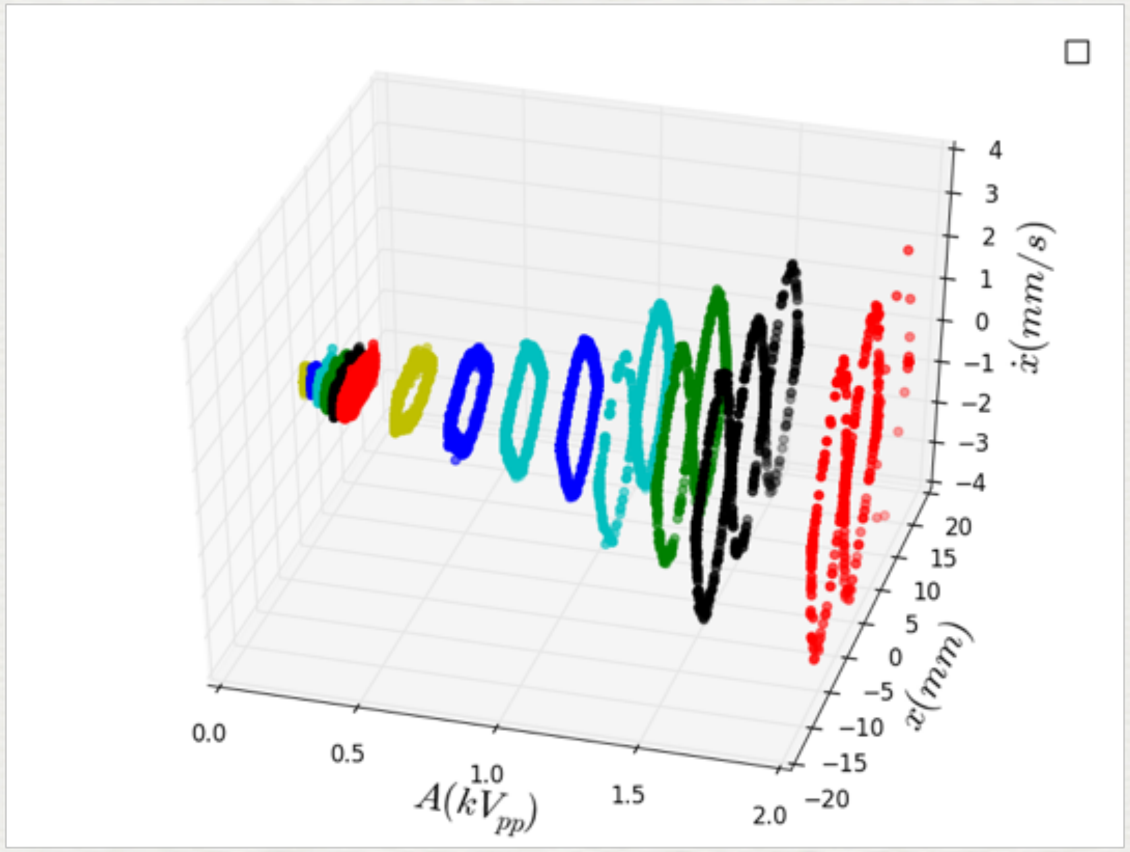
Equation of motion

$$\ddot{x} = A\sin x \cos t - \beta\dot{x}$$

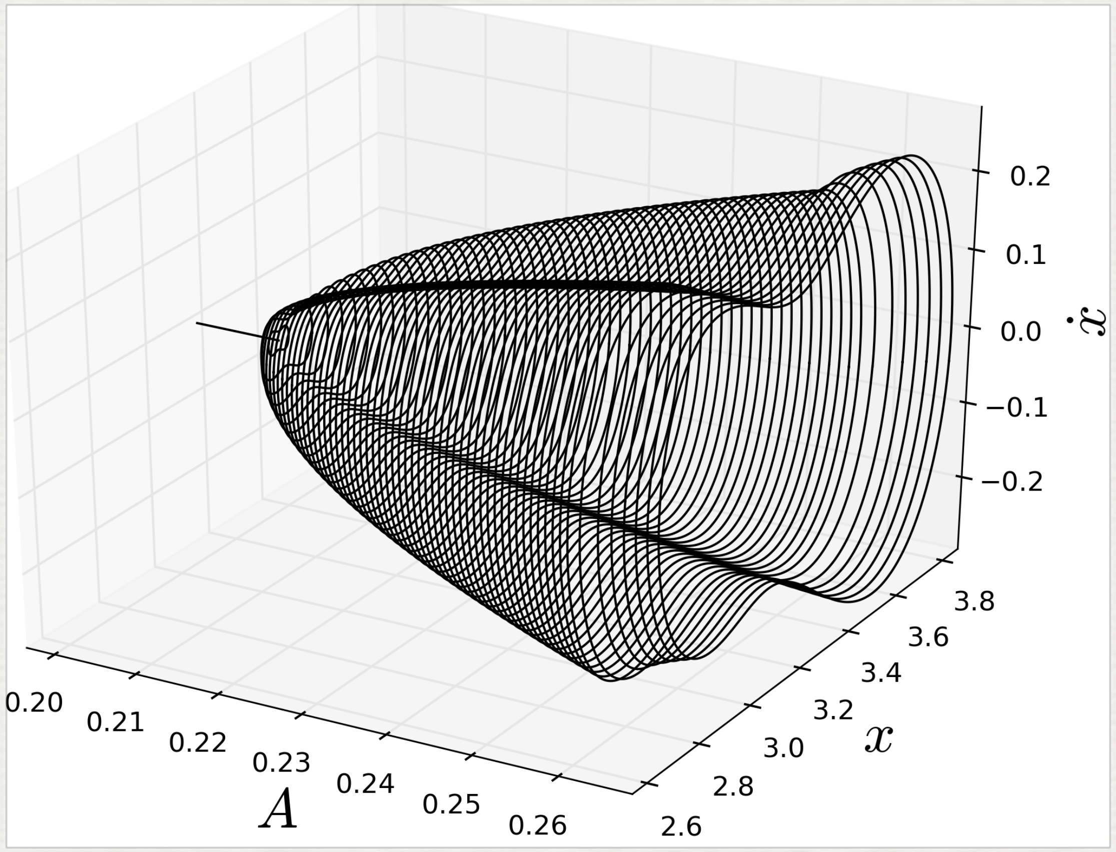
$$\beta = 0.6$$

First Single Particle Bifurcations

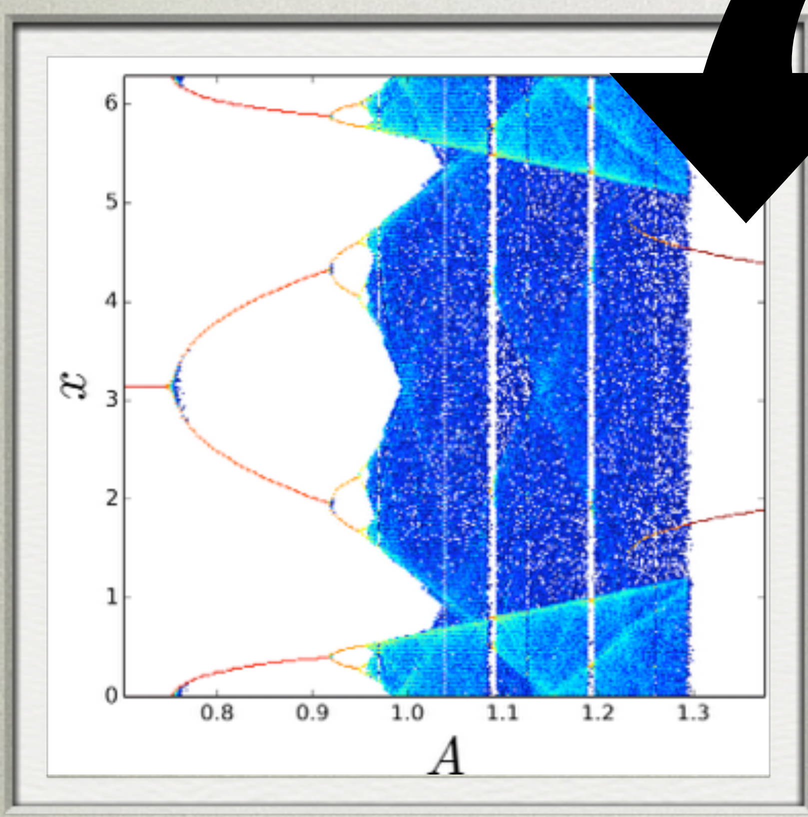
Experimental

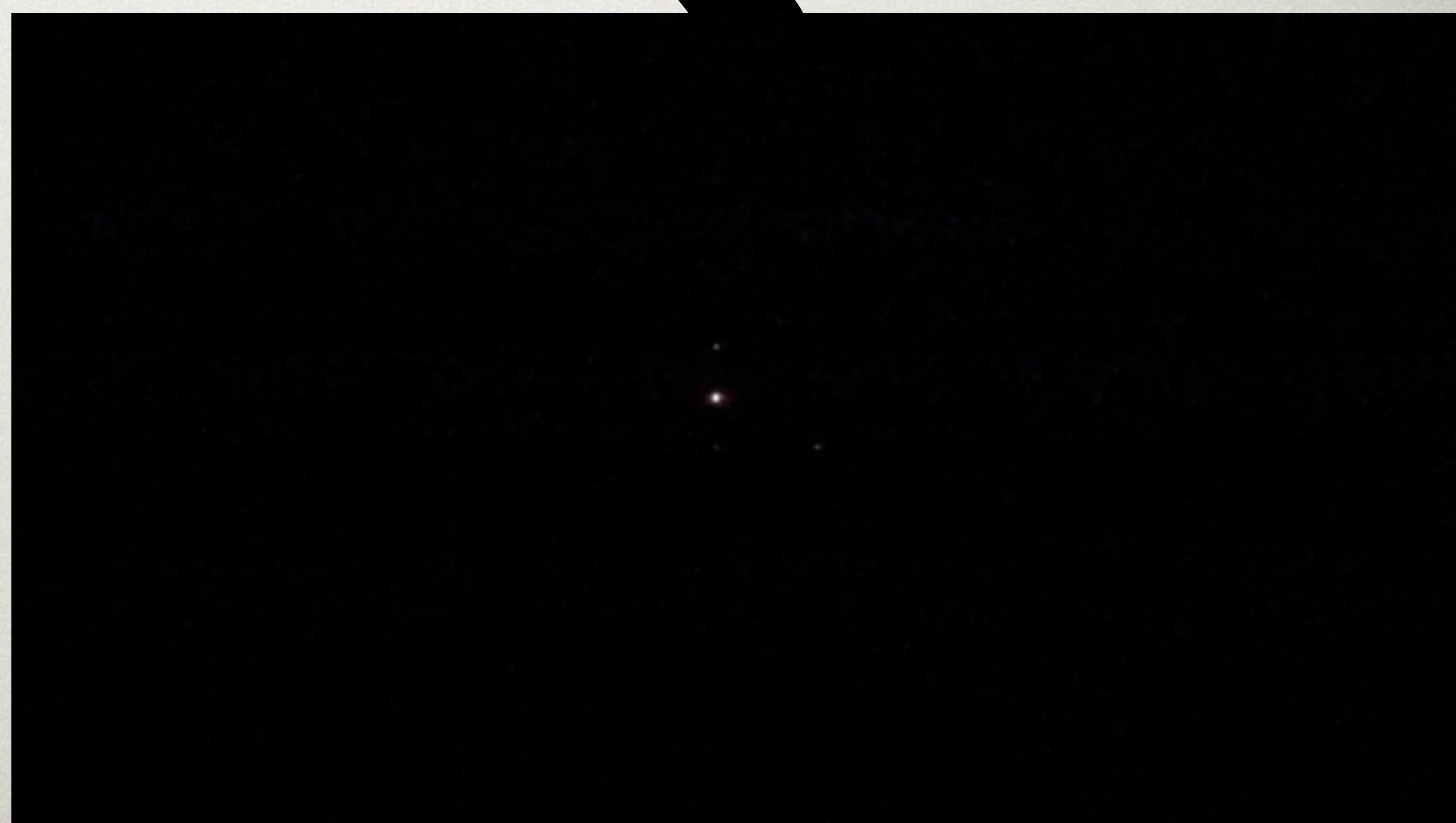


Numerical



Propagating Trajectory

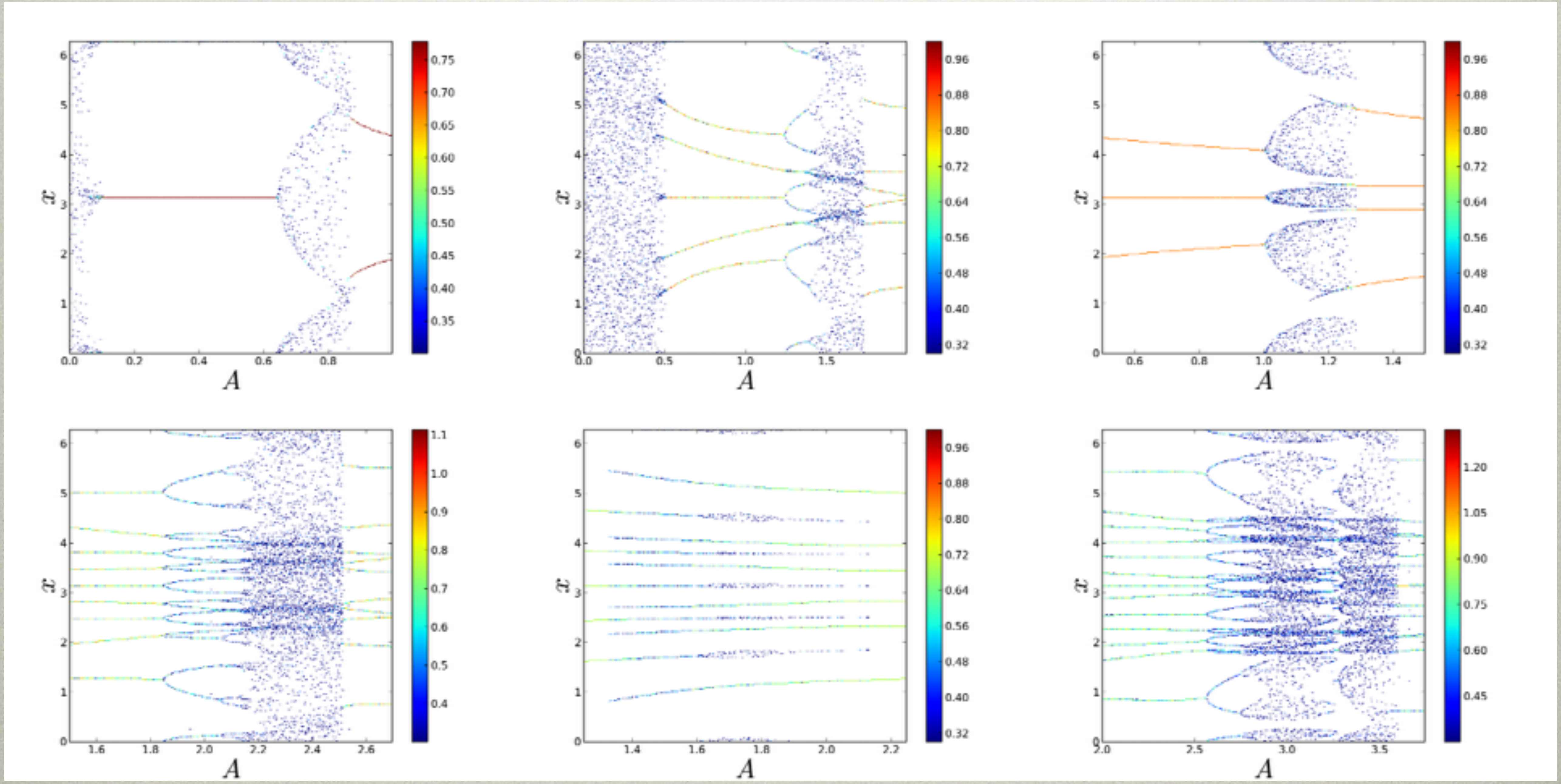




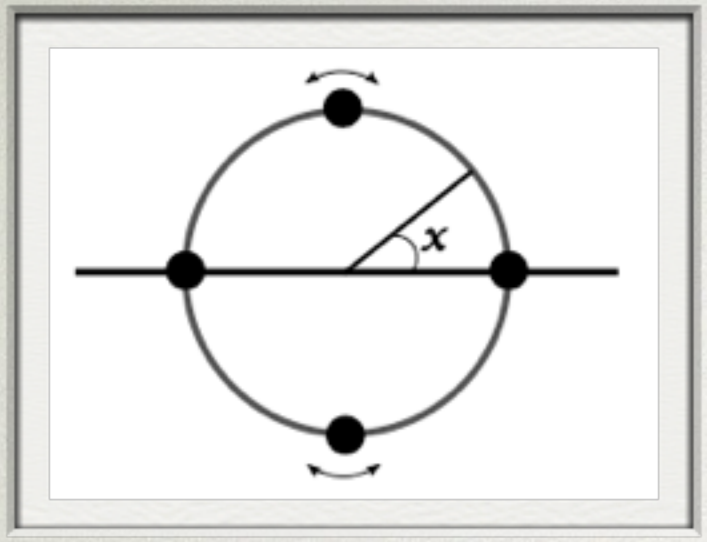
$q^2 = 1.0$

Multiple Particles (Integer Concentrations)

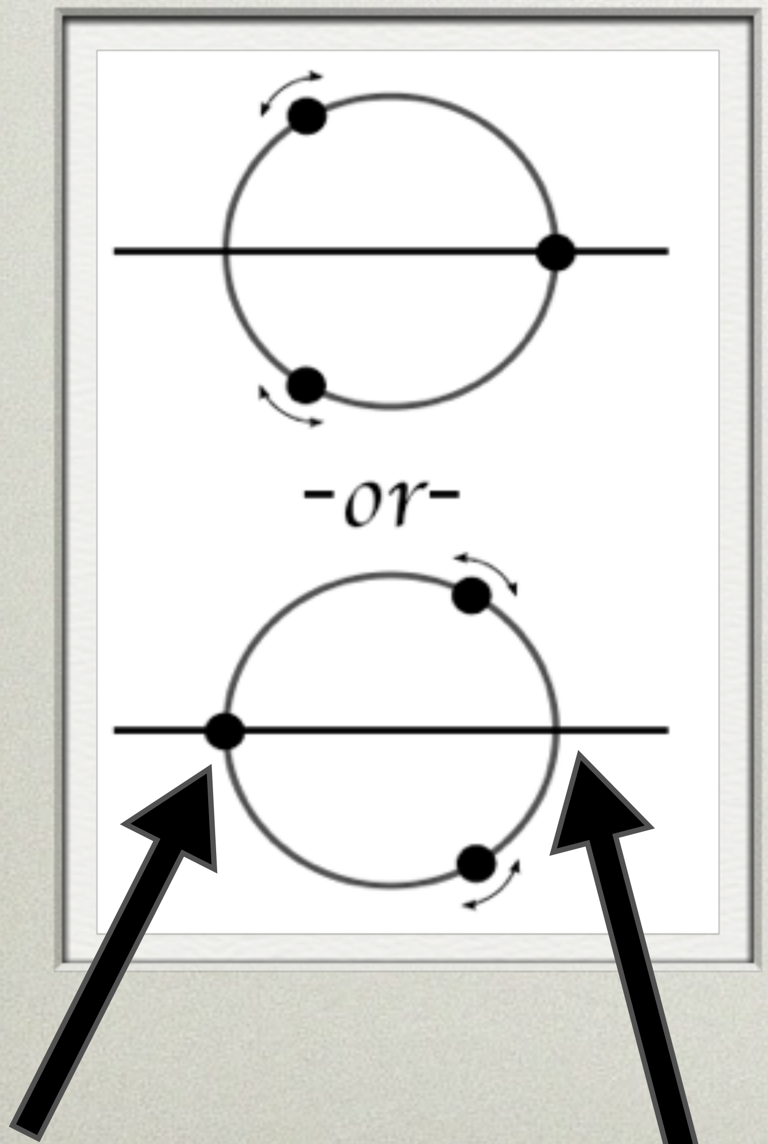
N=2 N=3 N=4



N=5 N=6 N=7



N=4

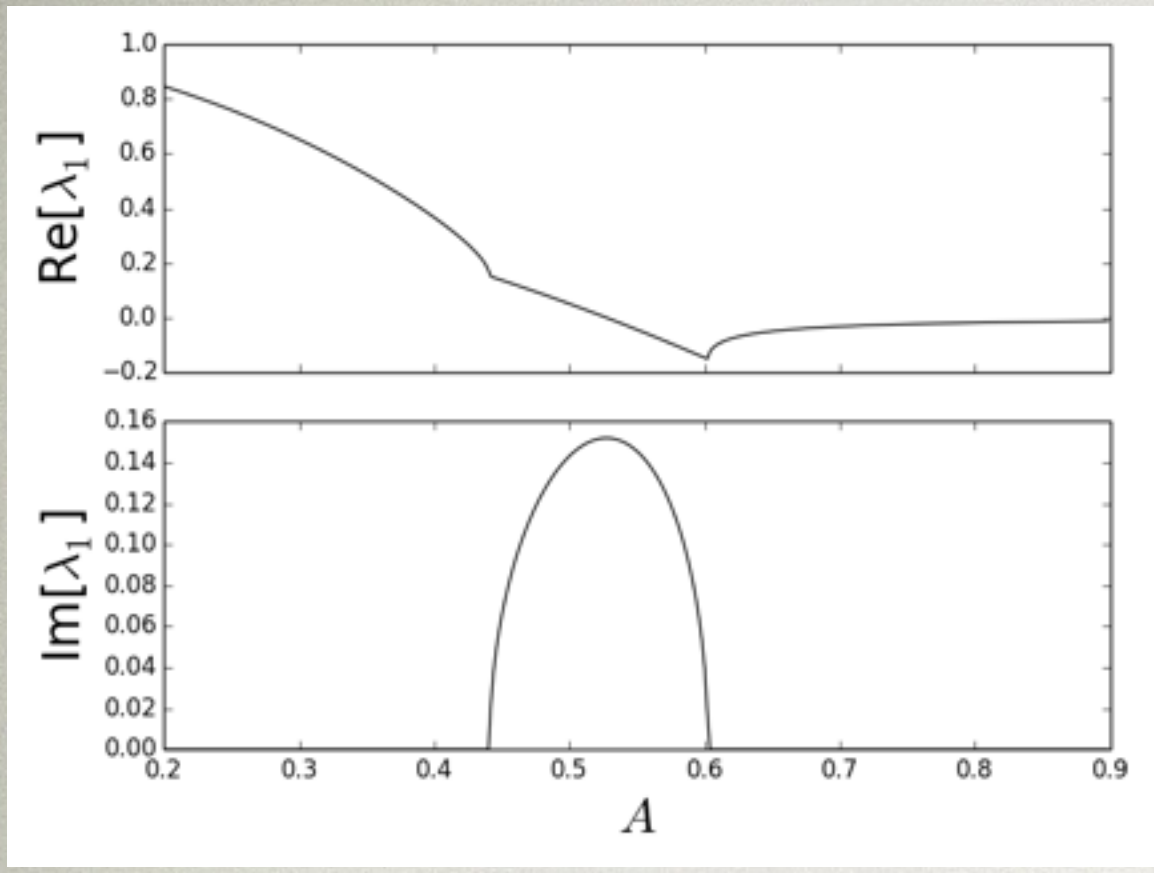


N=3

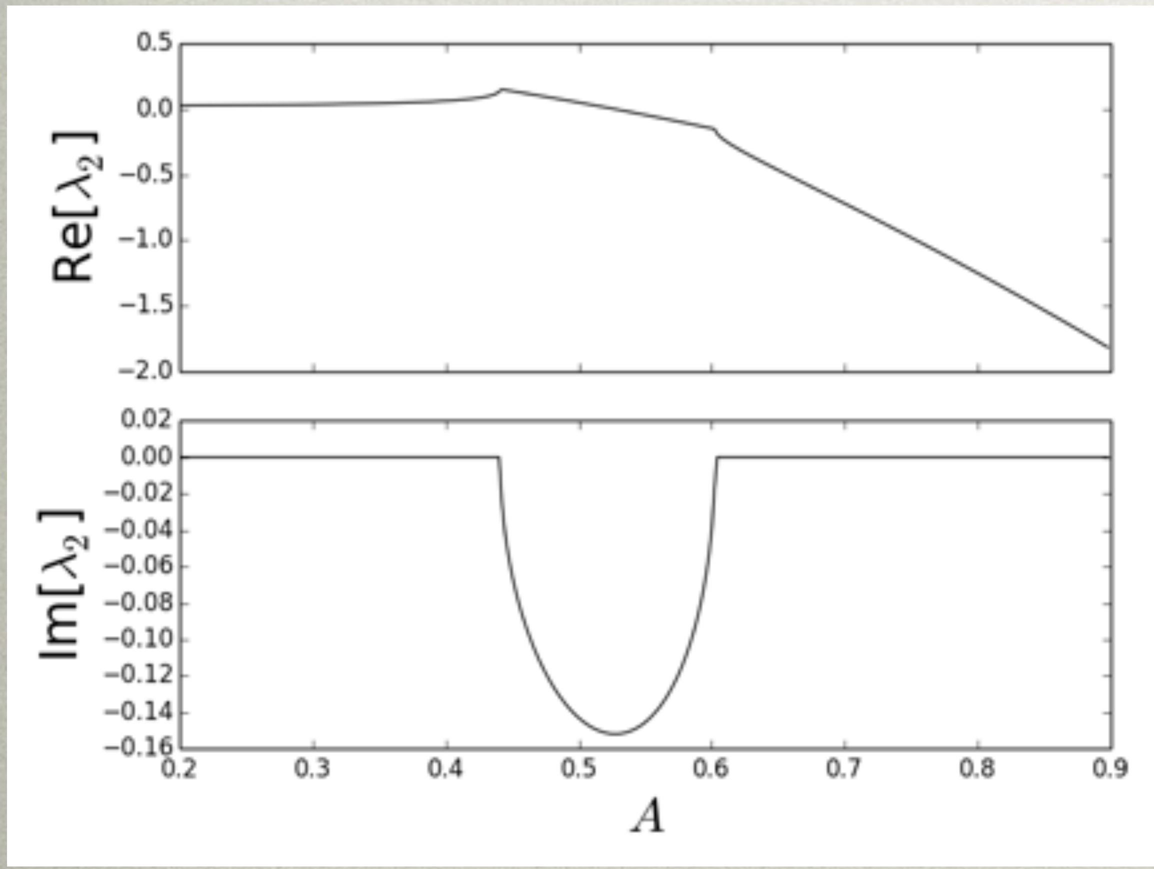
Antinode

For small *A* the time average force points in the direction of the antinodes of the potential

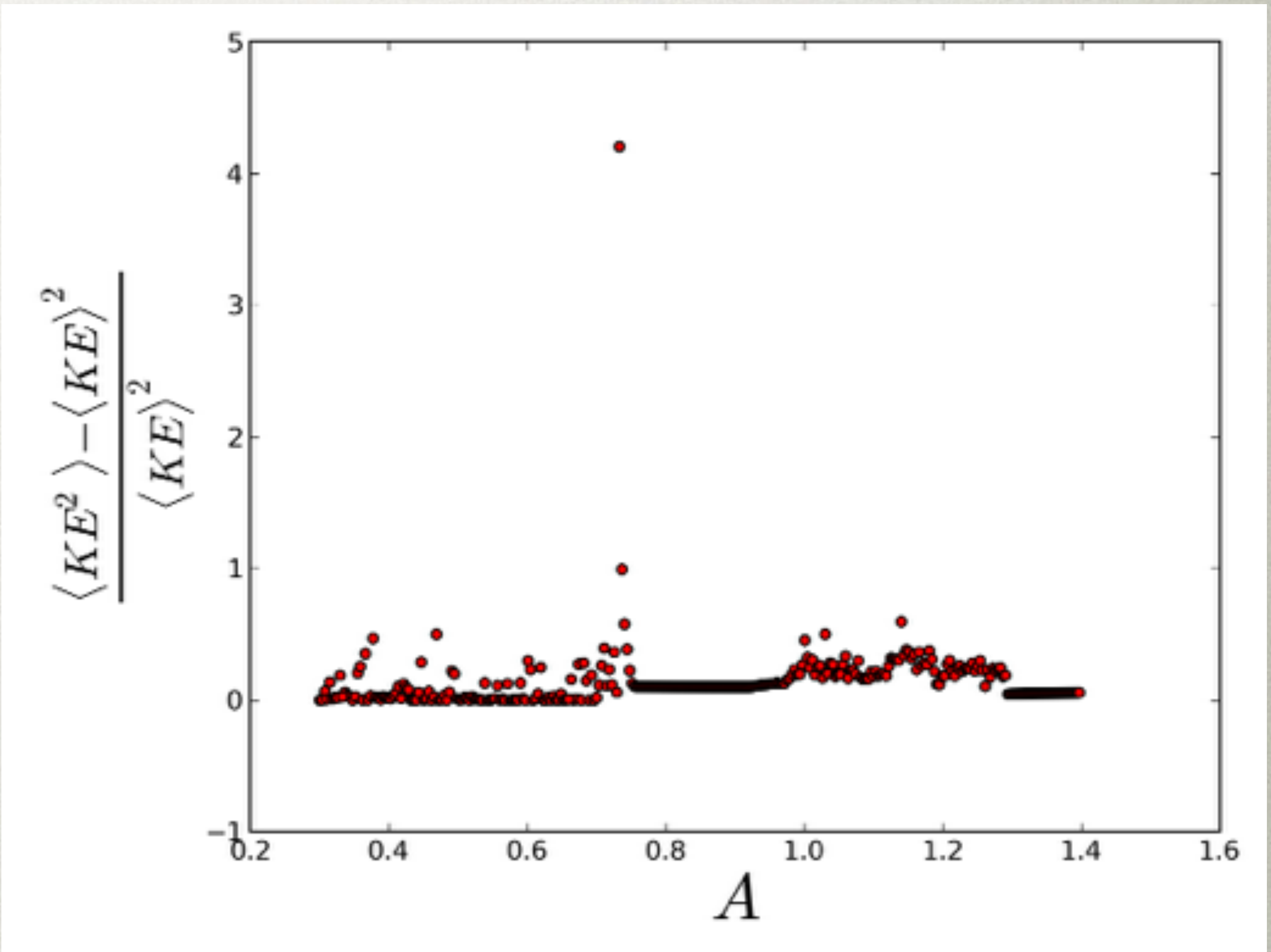
N=1



Re[λ_1]
Im[λ_1]
A



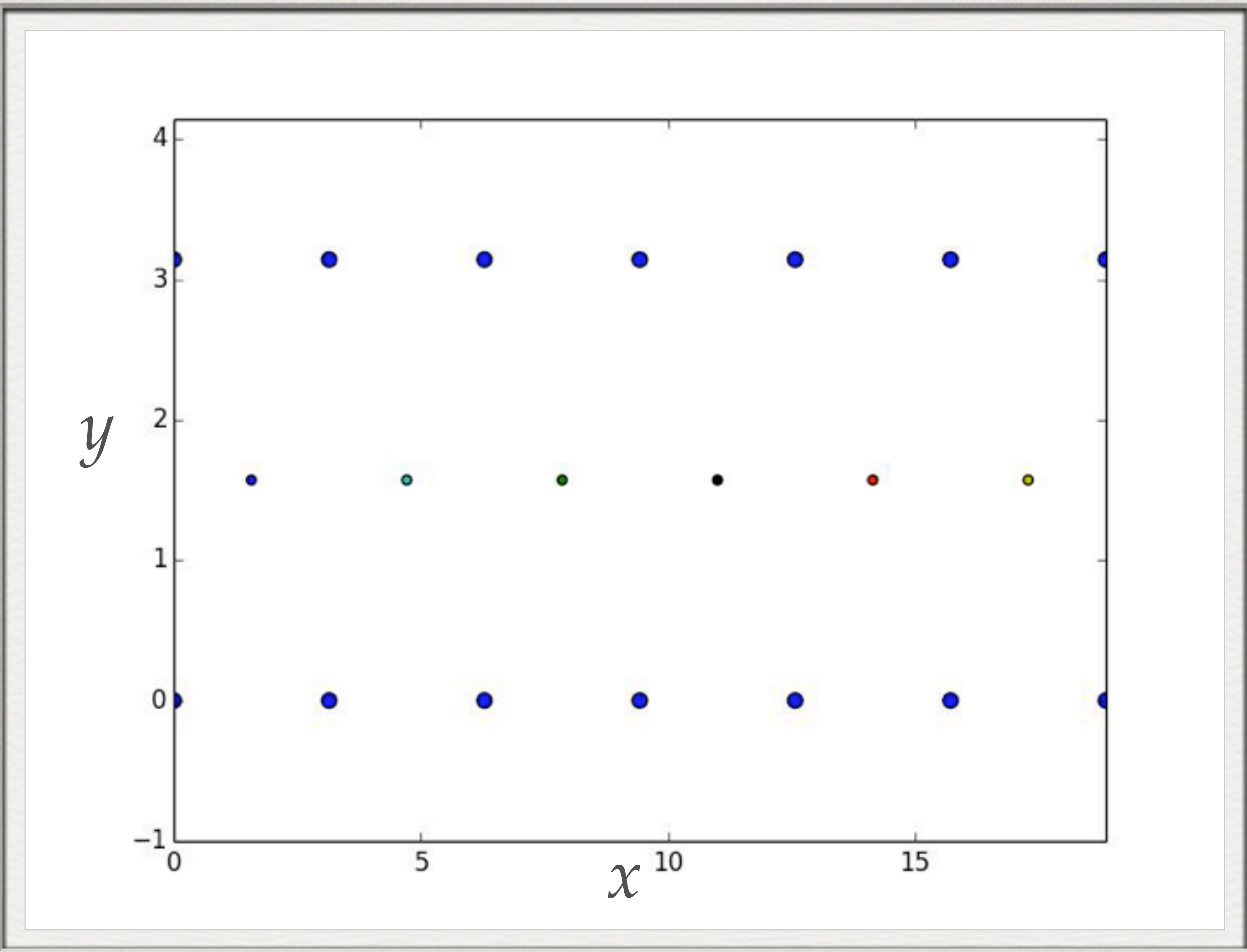
Re[λ_2]
Im[λ_2]
A

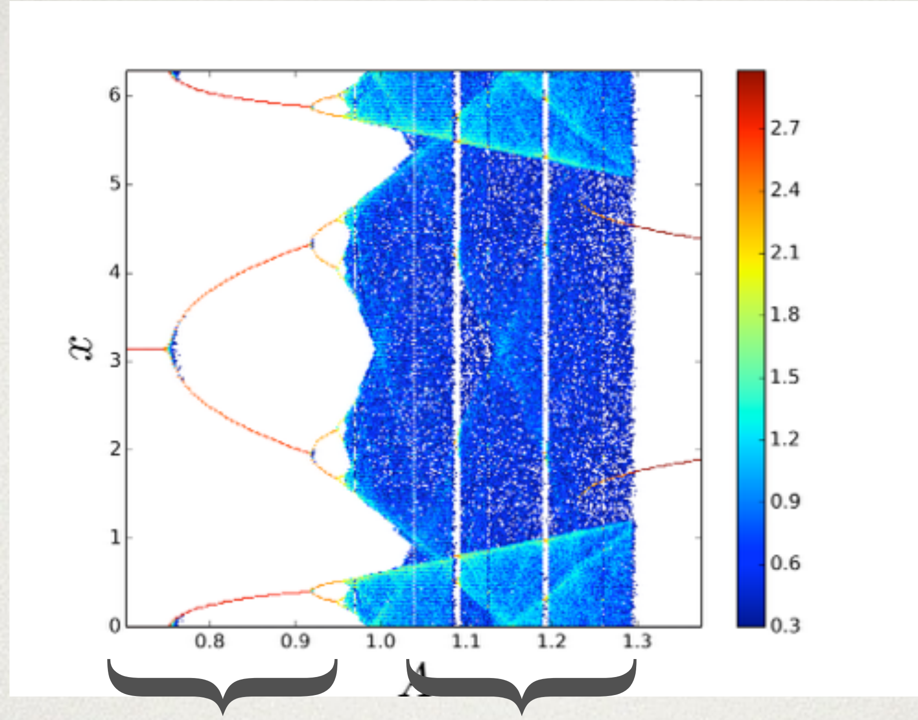


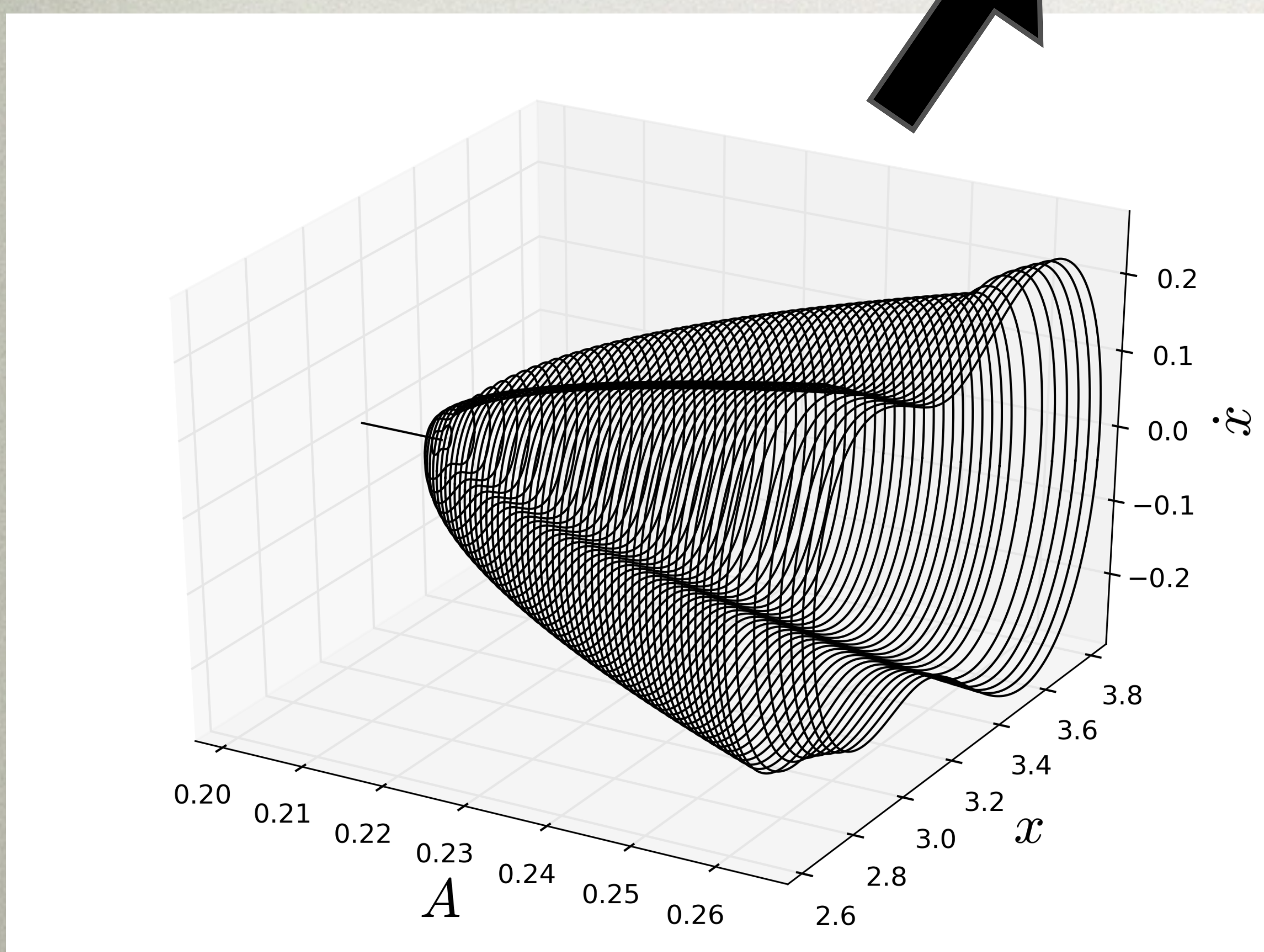
$\frac{\langle KE^2 \rangle - \langle KE \rangle^2}{\langle KE \rangle^2}$
A

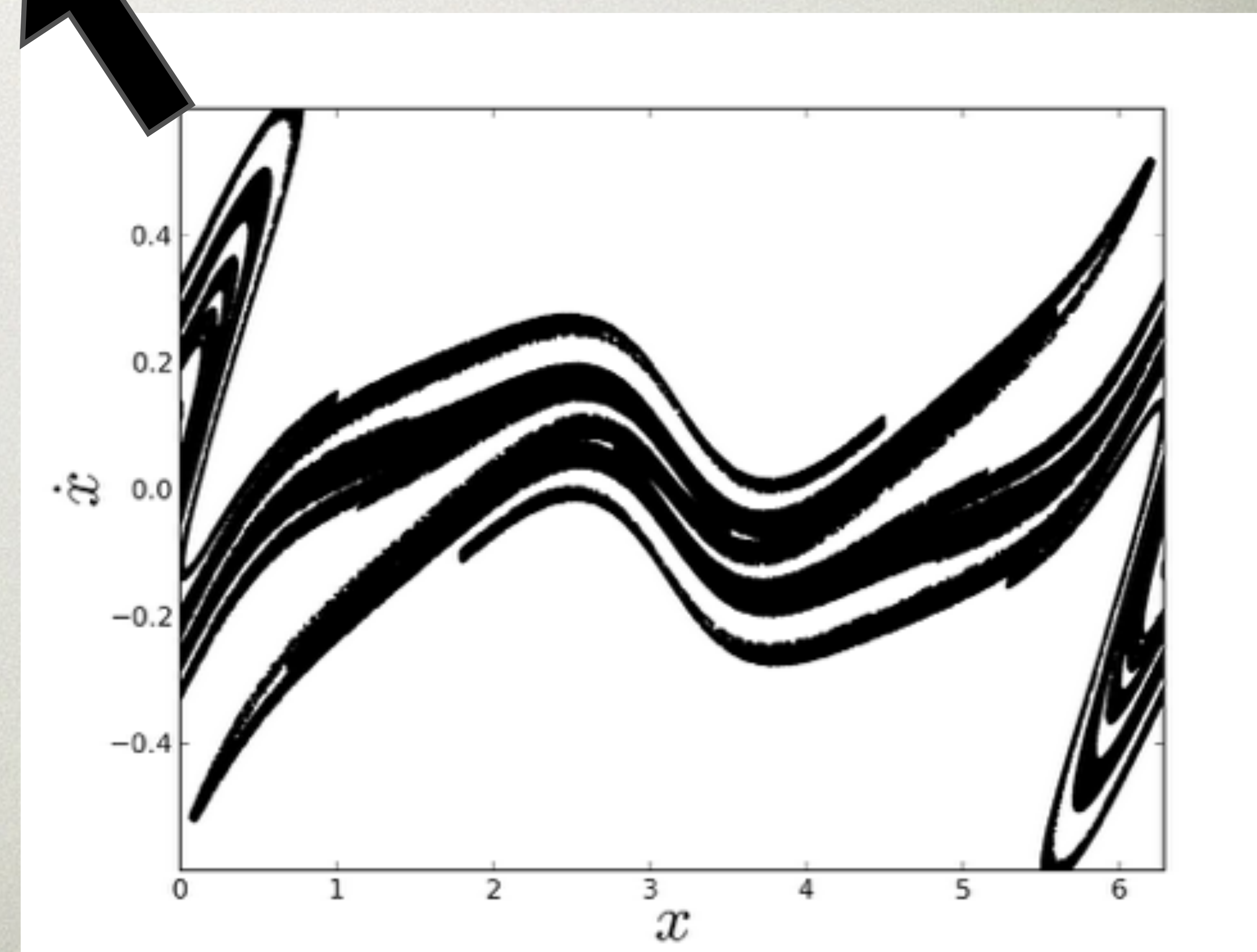
Multiple Particle Experiment (n=5)

Simulation (Periodic Boundary Conditions)

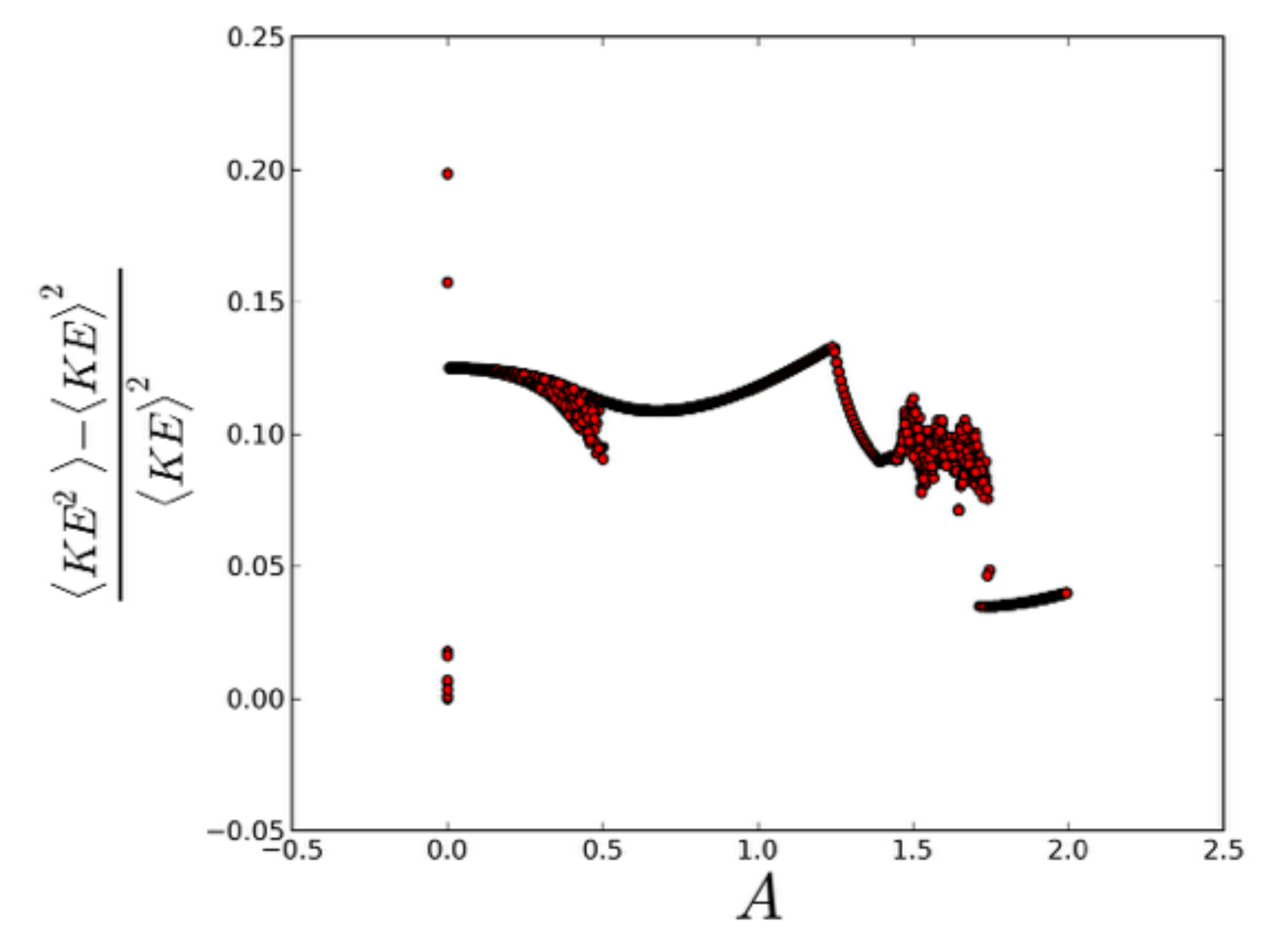



6
5
4
3
2
1
0
x
0.8
0.9
1.0
1.1
1.2
1.3
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6
0.3

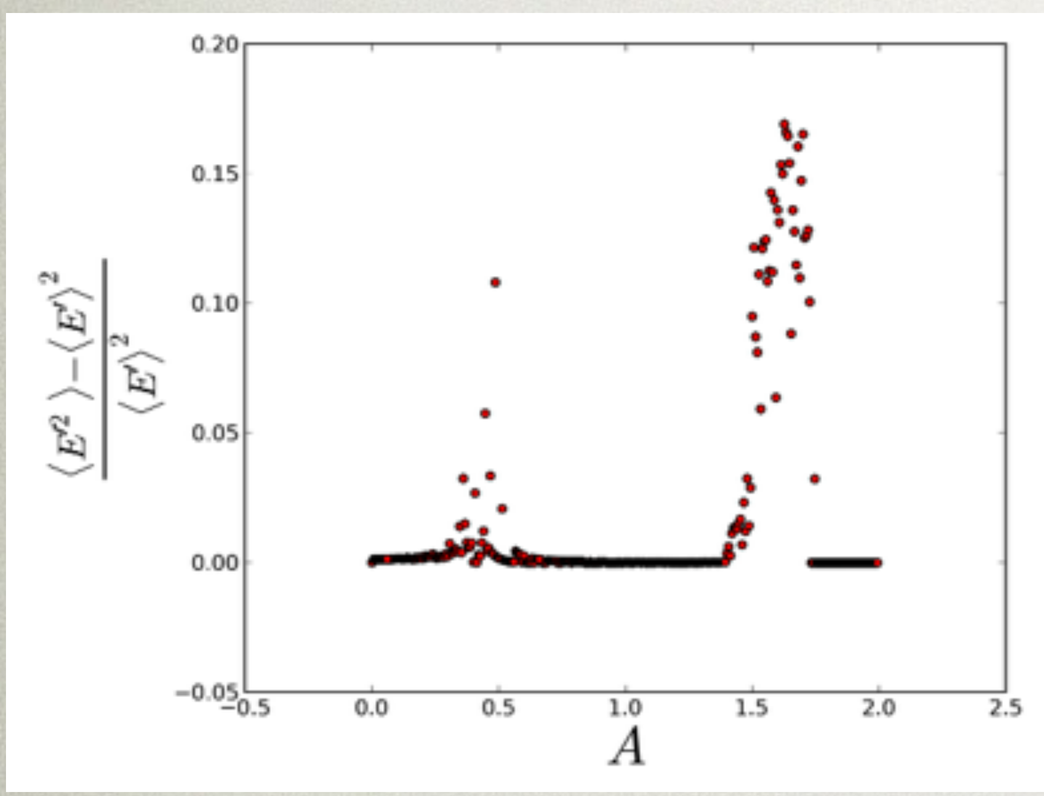

0.20
0.21
0.22
0.23
0.24
0.25
0.26
A
2.6
2.8
3.0
3.2
3.4
3.6
3.8
x
0.2
0.1
0.0
−0.1
−0.2
ẋ

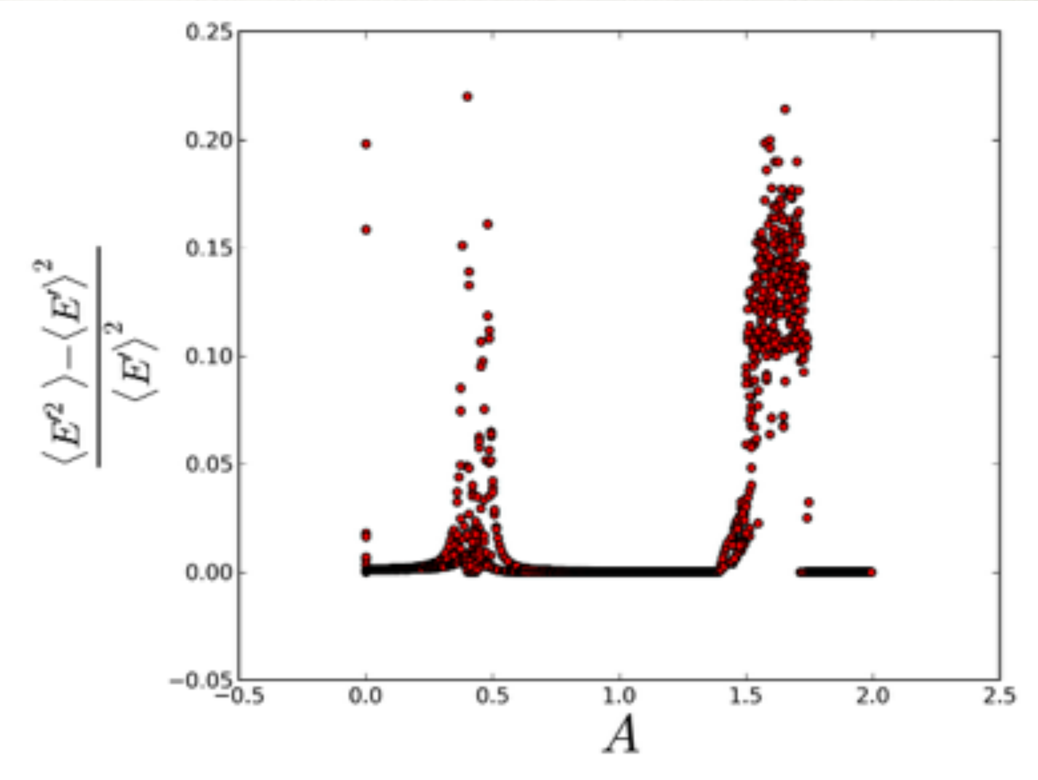

0.4
0.2
0.0
−0.2
−0.4
ẋ
0
1
2
3
4
5
6
x

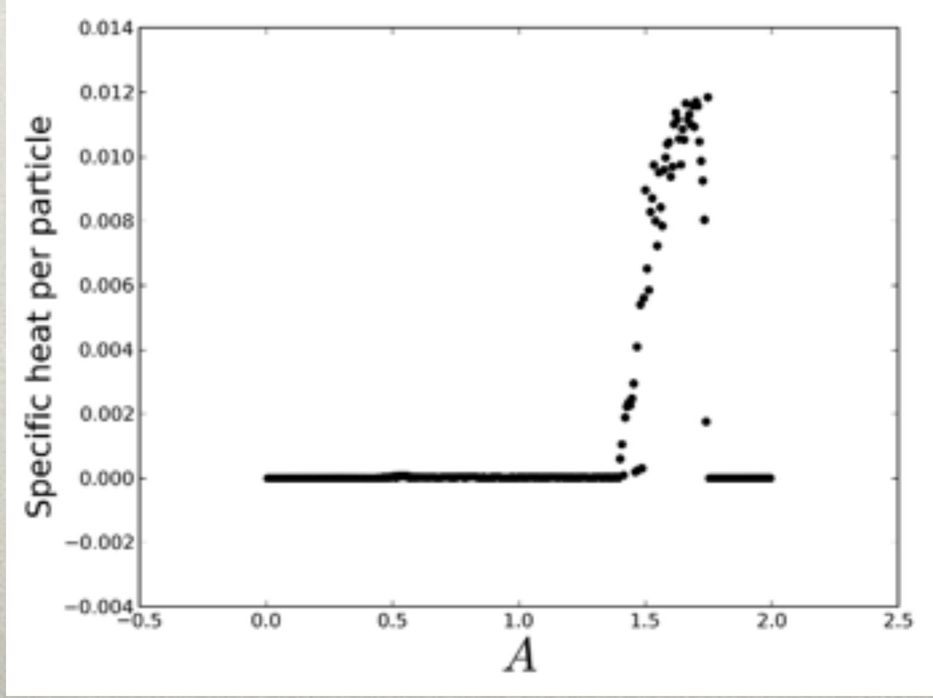
Not sliced
N=3 (avg. over 9 runs)



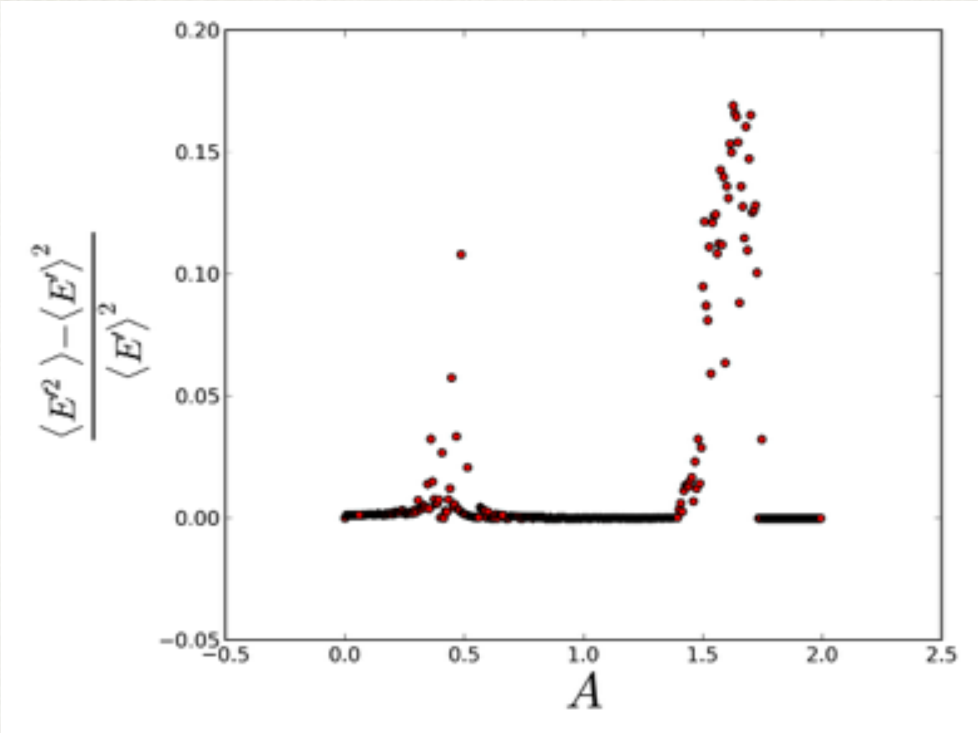
Sliced
N=3

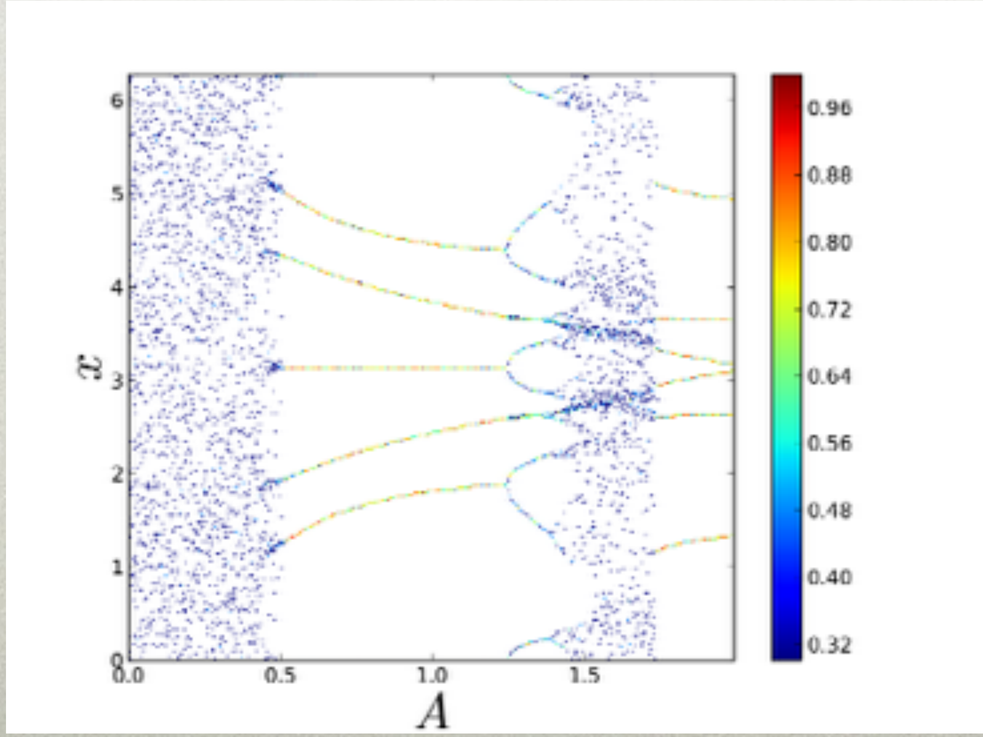

0.20
0.15
0.10
0.05
0.00
−0.05
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
$\frac{\langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle^2}$
A


0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
$\frac{\langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle^2}$
A

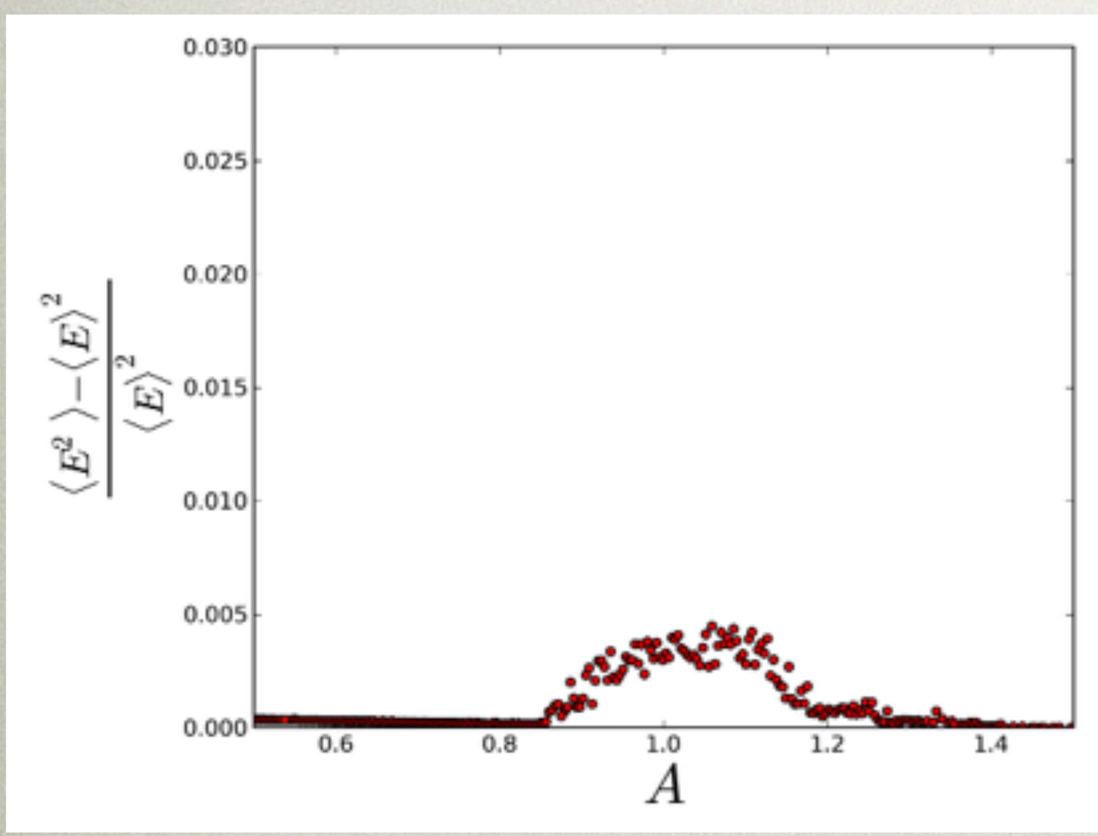

0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
Specific heat per particle
A

Sliced N=3

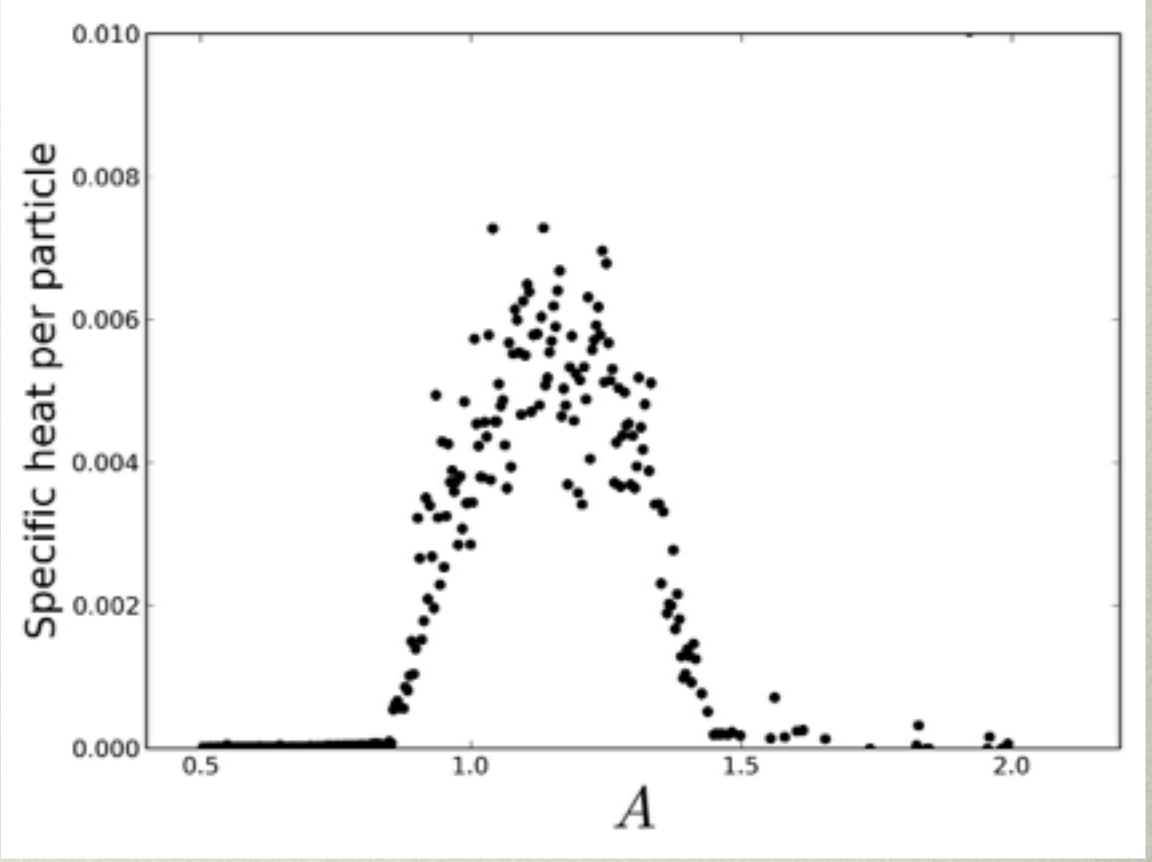

0.20
0.15
0.10
0.05
0.00
−0.05
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
A
$\frac{\langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle^2}$


0
1
2
3
4
5
6
x
0.0
0.5
1.0
1.5
A
0.96
0.88
0.80
0.72
0.64
0.56
0.48
0.40
0.32

N=20



0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.6
0.8
1.0
1.2
1.4
A
$\frac{\langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle^2}$



0.010
0.008
0.006
0.004
0.002
0.000
0.5
1.0
1.5
2.0
A
Specific heat per particle