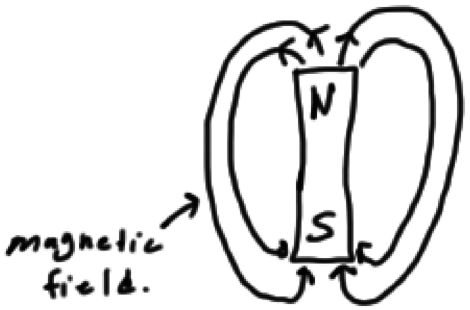
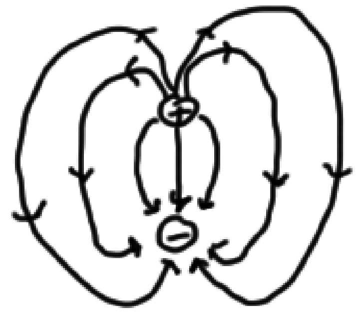
Magnetic forces and fields



looks similar to

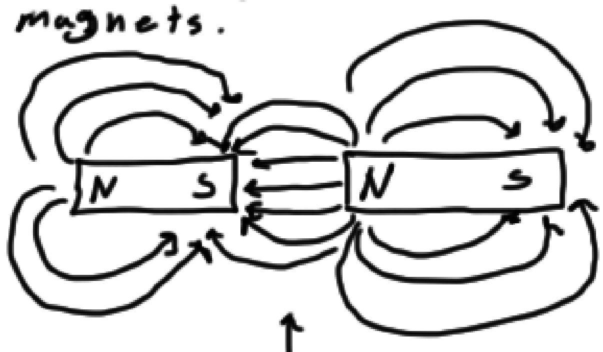


Its as if there are positive and negative magnetic charges at the north and south Poles of a magnet respectively.

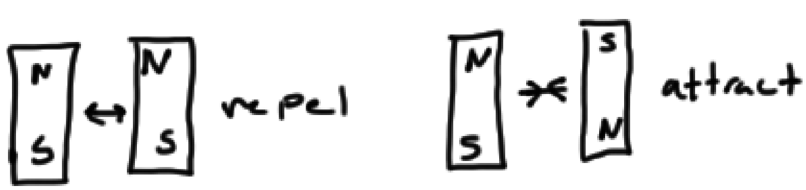
to make this idea stronger like Poles repel and opposites attract.

→ BUT we never find single magnetic charges (called monopoles) in nature.

→ cut a mag. in half → two smaller magnets.

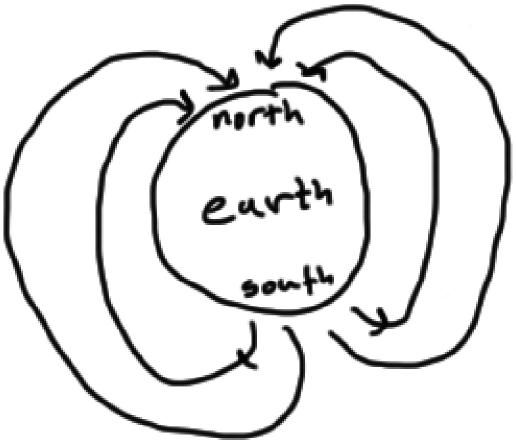


N attracted to south

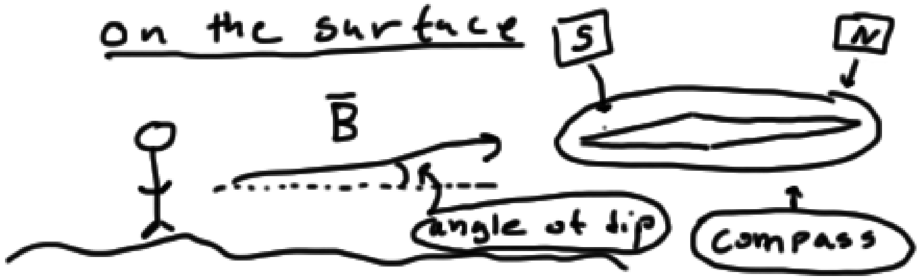


Magnets will try to align with an external field

→ this is how a compass works

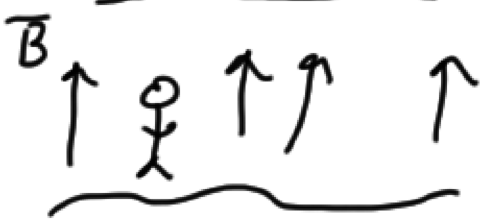


on the surface

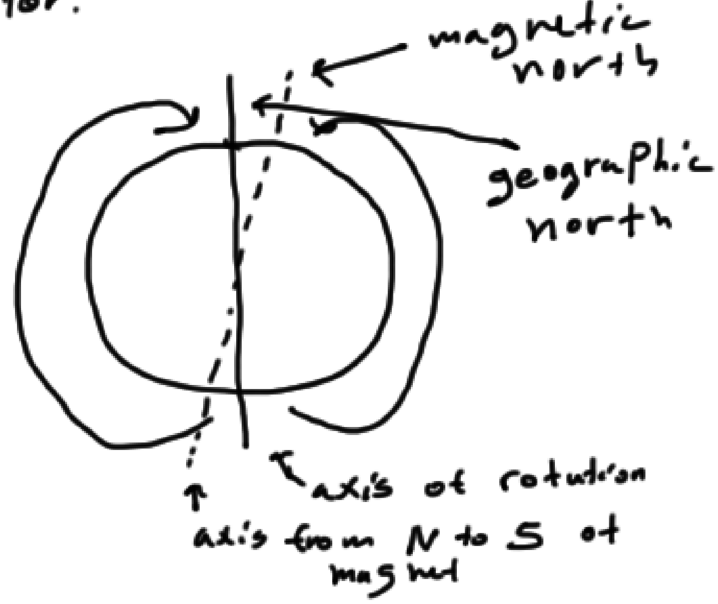


$\bar{B}$ almost parallel to surface near equator.

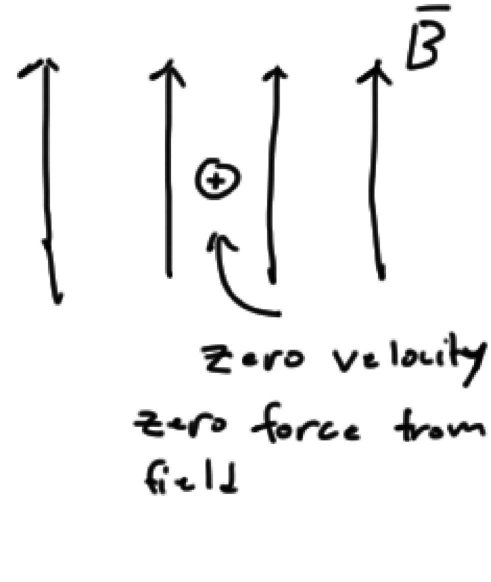
at South pole



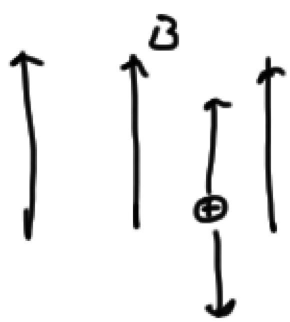
However



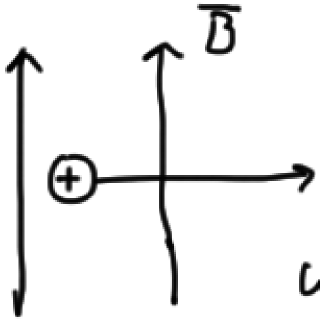
Why does earth have a magnetic field?
→ because it is rotating
→ we will understand this better as we move on.
For now know that the earths rotation causes charges to go in a giant circle → they create the $\bar{B}$ field.



Before discussing how moving charges "create" a $\bar{B}$ field we should define how moving charges move IN a $\bar{B}$ field



$\Rightarrow$ Velocity along $\bar{B}$
→ zero force from $\bar{B}$



Max force for velocity $\perp$ to $\bar{B}$

Which direction is $\bar{F}_B$?

→ also $\perp$ to $\bar{B}$.

In this case out of the page.

Can express all of this with a simple rule

$$\boxed{\begin{array}{c} \bar{F} = q(\bar{V} \times \bar{B}) \\ \hline |F| = q\,|\bar{V}|\,|\bar{B}| \sin\theta \end{array}}$$

angle between $\bar{B}$ and $\bar{V}$.

What about direction? → use right hand rule

What are units of $\bar{B}$? → Tesla (T)

T? → write force eqn. as $|B| = \frac{|F|}{|qV| \sin\theta}$

→ T is in $\frac{\text{Newton} \cdot \text{sec}}{\text{Coulomb} \cdot \text{meter}}$

<u>Some examples</u>

What is $\bar{F}_B$ if $\bar{V} = 2 \frac{m}{s} \hat{x}$ $\quad \bar{B} = 1T \hat{y}$

$$\bar{F}_B = q\bar{V} \times \bar{B} = q \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = q\left[\hat{x}(0) - \hat{y}(0) + \hat{z}(2 \times 1)\right] = q2\hat{z}$$

$$\boxed{\bar{F}_B = q2\hat{z}}$$

What if the charge was ⊖ → $\bar{F}_B = -q2\hat{z}$

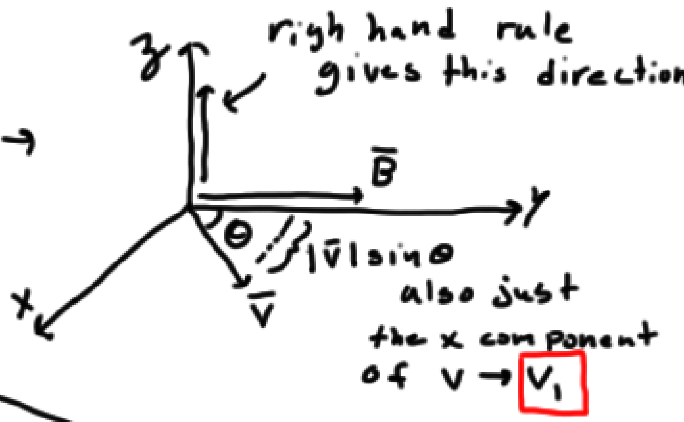
Just get opposite direction

For $\overline{V} = V_1\hat{x} + V_2\hat{y}$ and $\overline{B} = B\hat{y}$ and q given, what is $\overline{F}_B$?

$$\overline{F}_B = q(\overline{V}\times\overline{B}) = q\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ V_1 & V_2 & 0\\ 0 & B & 0\end{vmatrix} = q\left[\hat{x}(0) - \hat{y}(0) + V_1 B\hat{z}\right] = \boxed{qV_1B\hat{z}}$$

→ Same direction as befor?

Do a different way. Find direction with right hand rule. Then see what $q|V|\sin\theta$ is } → Draw →



so $|\overline{v}|\sin\theta = V_1$

Now $|\overline{F}_B| = q|\overline{v}||\overline{B}|\sin\theta$

$$= q|\overline{B}||\overline{V}|\sin\theta = \boxed{qBV_1}$$

right hand rule gave us direction $+\hat{z}$

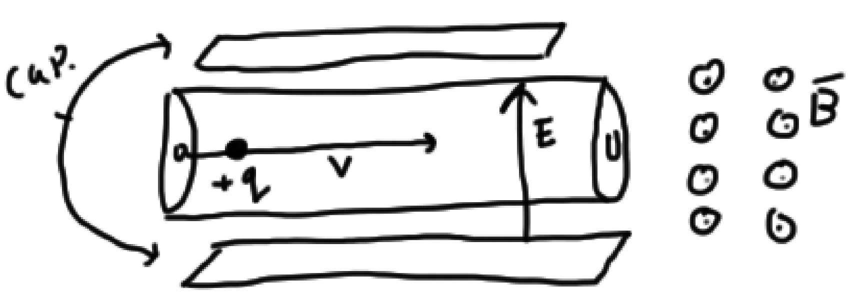
so $\boxed{\overline{F}_B = qBV_1\hat{z}}$

We can combine the electric and magnetic forces into one nice expression

$$\overline{F} = q(\overline{E} + \overline{v}\times\overline{B})$$

By using $\overline{B}$ and E we can select particles with certian velocities.

Blance E & B in the picture below



$q\overline{E} = V\times\overline{B}$ have the directions

so

$qE = VB$ $\boxed{V = \frac{qE}{B}}$

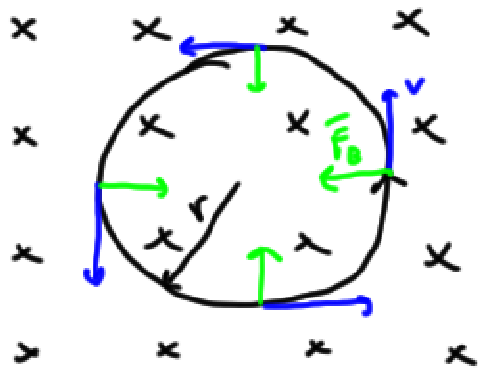
Does $\bar{B}$ do work?

Remember that work $W = \bar{F} \cdot \bar{D}$

$\bar{F} \cdot \bar{D} = FD\cos\theta$ so is $\bar{F}$ is always $\perp$ to D then no work is done.

$\bar{F}_B$ IS always $\perp$ to $\bar{V}$ so $\boxed{W_B = 0}$

Since this is true it can be simular to a ball on a string.



- No work is done
- the direction of $\bar{V}$ changes but not $|V|$
- $\bar{F}_B$ always $\perp$ to $\bar{V}$

Centripetal force $\rightarrow F_c$

$$F_c = \frac{mv^2}{r}$$

In this case F_B is F_c

i.e. $\bar{F}_B = \bar{F}_c$

$$qVB = \frac{mV^2}{r}$$

given $\bar{B}$ and $\bar{V}$ what would r be?

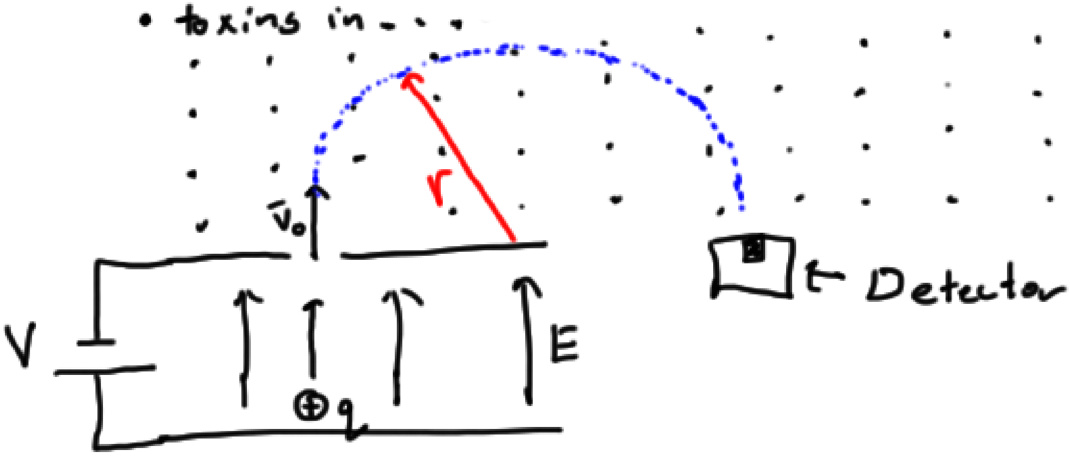
$$\boxed{r = \frac{mV}{qB}}$$

Mass Spectrometer

used to analyze the amount of a particular molecules or atoms in some mixture.

eg

- anesthesiologist → check gas mixtures.
- drug preparation
- pesticides
- toxins in . . .



if many charged bodies are coming through the Detector can sweep back and forth counting how many bodies enter it at any given position

← The position and size of peaks tell you what chemicals are present

Current in magnetic field

Not so different that one particle.
A current through a wire has many

Know that

$$|\bar{F}_B| = q|\bar{v}||\bar{B}|\sin\theta$$

units → charge $\frac{m}{s}$ T

current → $I = \frac{\Delta q}{\Delta t}$ → $\frac{\text{charge}}{\text{time}}$

so $I|\bar{B}|\sin\theta$ → almost has units of force

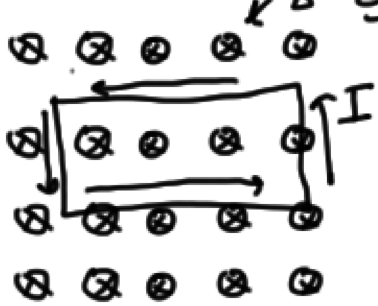
units → $\frac{c}{s}$ T → missing length (m) → this is length of wire

$$\boxed{I L |\bar{B}| \sin\theta = |\bar{F}_B|}$$

we could also write as cross product

$$\boxed{L\bar{I} \times \bar{B} = \bar{F}_B}$$

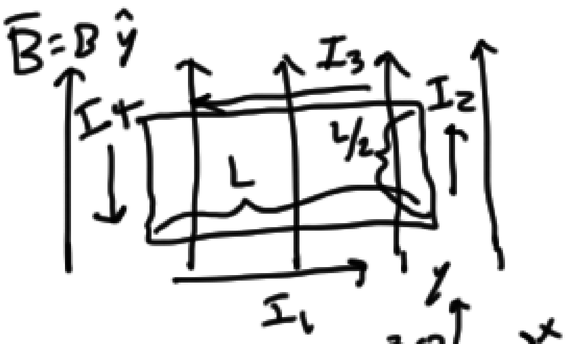
Ex



← $\bar{F}$ on each piece is inward

→ Net $\bar{F} = 0$

What about:



on I_1: $F_B = I_1 L B \overbrace{\sin\theta}^{1} \hat{z}$

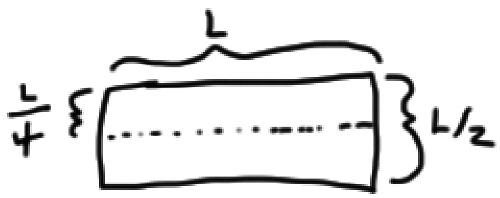
on I_2: $F_B = I_2 \frac{L}{2} B \overbrace{\sin\theta}^{0} = 0$

on I_3: $F_B = I_3 L B \overbrace{\sin\theta}^{1} (-\hat{z})$

on I_4: $F_B = I_4 \frac{L}{2} B \sin\theta = 0$

Last ex. showed that for some orientations there is torque on a loop in a magnetic field with a current through it.

for our loop



$$\tau = \bar{r} \times \bar{F} = rF\sin\varphi\,(-\hat{x})$$

$\varphi \to$ angle between $\bar{r}$ & $\bar{F}$

$$\tau = ILB\left(\tfrac{L}{2}\right)\underbrace{\sin\left(\tfrac{\pi}{2}\right)}_{1}$$

$$\tau = \underbrace{(L)\left(\tfrac{L}{2}\right)}_{\text{Area !}} I B$$

$$\tau = AIB\sin\varphi$$

$\rightarrow$ or

This area is the area of the loop for any shape

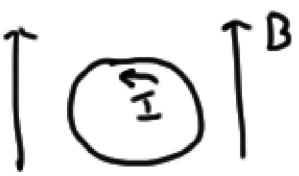
<u>ex</u>

Circle loop

here it is 0

$\tau = IAB\sin\varphi$ so $\boxed{\tau = 0}$

<u>ex</u>



$\pi/2$

$\tau = IAB\sin\varphi$

$\boxed{\tau = IAB}$

which direction will it rotate?

$\rightarrow$ Just look at top of loop and pretend it is straight.

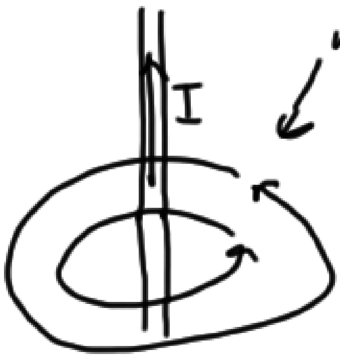
$\bar{F}$ on top is going into the paper so the top would rotate away.

What if the wire coils around twice to make two loops?

Then τ is $2 \times \tau$ of single

More generaly then $\boxed{\tau = N I A B \sin\varphi}$

Magnetic fields produced by currents



use right hand rule to figure out direction.

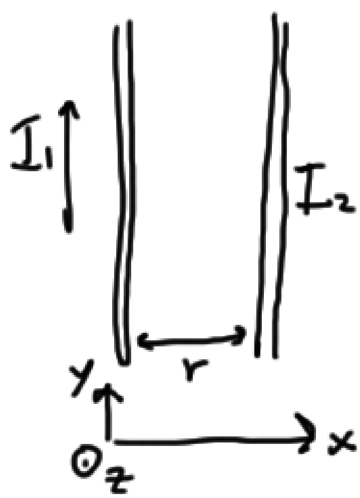
$$B = \frac{\mu_0 I}{2\pi r}$$

μ_0 ← has to do with light which communicates force

r ← dist from wire

2π ← has to do with circles

Since I creates $\bar{B}$ then two currents must exert forces on one another.



B from $I_1 \Rightarrow \quad B = \frac{\mu_0 I_1}{2\pi r}$

$\bar{F}$ on I_2 is then

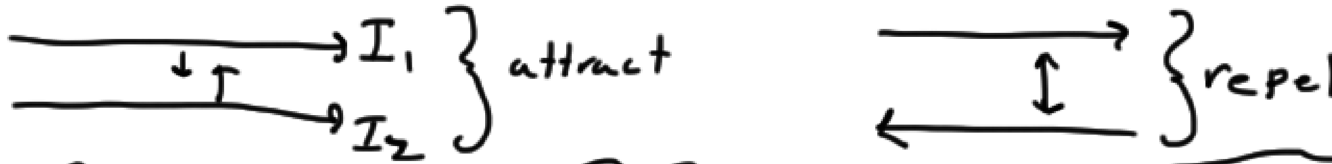
$|\bar{F}_B| = I_2 L B \sin\theta$ ← B is going into paper if both I are in xy plane

$$|\bar{F}_B| = I_2 L \frac{\mu_0 I_1}{2\pi r}$$

which direction is $\bar{F}_B$?
use right hand rule

$$\boxed{\bar{F}_B = I_2 L \frac{\mu_0 I_1}{2\pi r}(-\hat{x})}$$

so



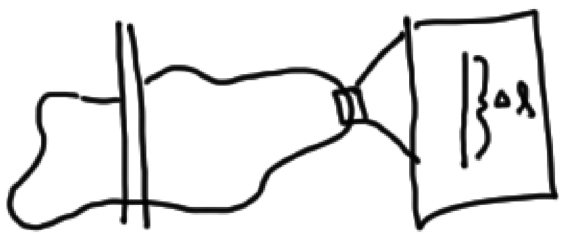
Ampere's Law (not on exam)

→ like Gauss's law but for current

Relates magnetic field to geometry of wire

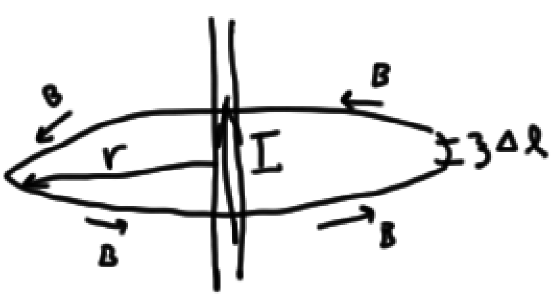
$$\sum B_{\parallel} \Delta l = \mu_0 I$$

part of $\bar{B}$ that is parallel to Δl



We should be able to get back what we already know about $\bar{B}$ from a wire.

$\bar{B}$ is parallel to Δl everywhere



so

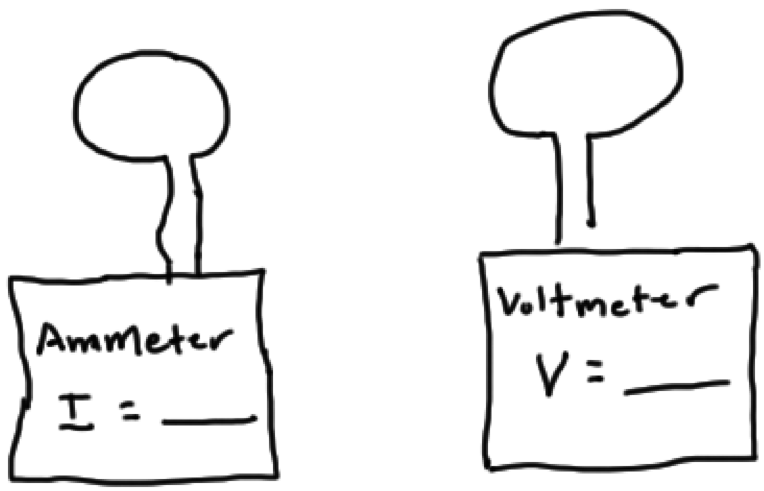
$$\sum B_{\parallel} \Delta l = \mu_0 I$$

becomes

$$B \underbrace{(2\pi r)}_{\text{circumference}} = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Electromagnetic Induction



If there in NO magnetic field passing through the loops, (zero flux)

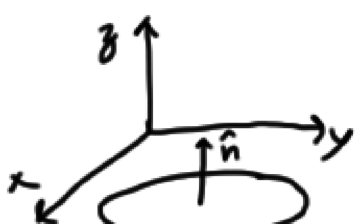
OR

if there is a CONSTANT magnetic field passing through the loops then both the Volt and ammeter will read 0. (constant flux)

However if the flux of the magnetic field through the loop depends on time than both the volt and ammeter will read non zero!

The definition of mag. flux is not much different than electric flux.

$$\Phi_B = BA\cos\phi$$ where ϕ is angle between $\hat{n}$ and $\bar{B}$



circle in xy plane

$\rightarrow \hat{n} = \hat{z}$

If $\bar{B} = B\hat{z}$ then $\Phi_B = B\pi r^2$

How can we get time dependent flux $\Phi(t)$?

- → rotate loop
- → move it out of $\bar{B}$ field.
- → introduce time dependence in $\bar{B}$
  - → rotate $\bar{B}$
  - → change magnitude of B
- → Chang area of loop

can see all of these by looking at Φ_B eqn.

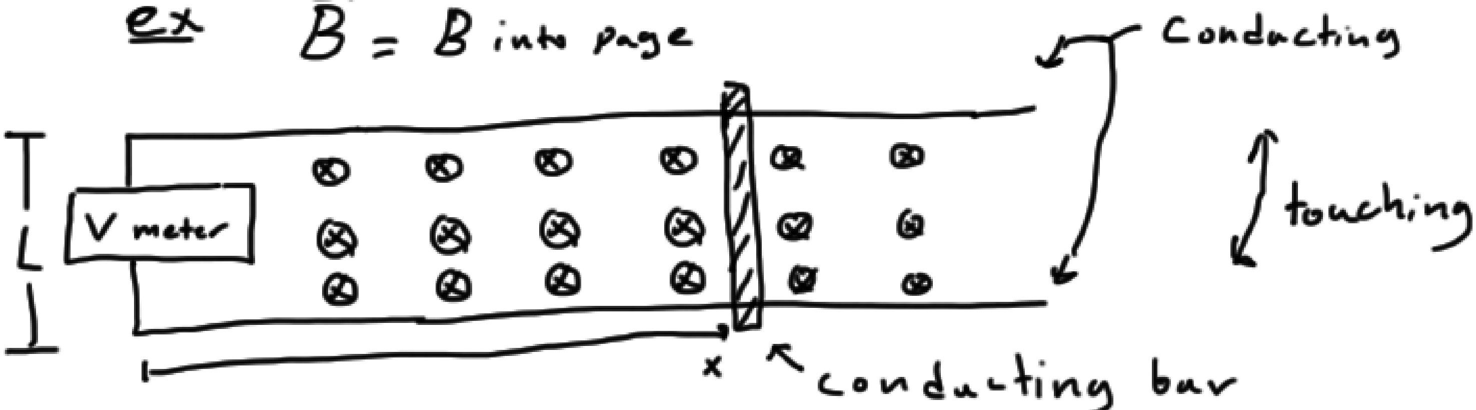
How is the induced current and Voltage related to the flux?

$\mathcal{E} \rightarrow \underbrace{\text{electromotive force}}_{\text{emf}}$ (realy just a voltage)

Induced $\mathcal{E}$:

$$\boxed{\mathcal{E} = \frac{\Delta \Phi}{\Delta t}}$$

<u>ex</u> $\bar{B} = B$ into page



Slide the bar towards Voltmeter. What does V meter read?

Given: B, L, v ← velocity of bar

lets say at $t=0$ the bar is at x

units of Velocity $\frac{m}{s}$ so it

$A^{\text{rea}}_{\text{at } t=0} = Lx \qquad A_{\text{at } t=1s} = L(x - v(1s))$

$\Phi_{t=0} = BLx\underbrace{\cos\phi}_{1} \qquad \Phi_{t=1} = BL(x - v1)$

$\Delta\Phi = BLx - BL(x - v1) = BL(x - x + v1) = BLv1_s$

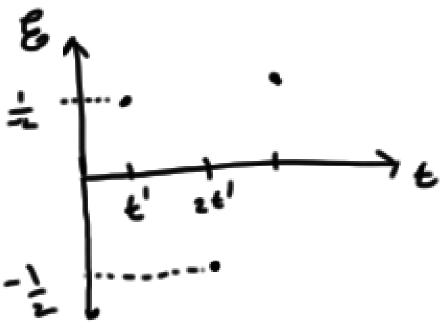
$\frac{\Delta\Phi}{\Delta t} = \frac{BLv1_s}{1_s} = BvL \qquad \boxed{\mathcal{E} = BvL}$

Rotating coil

$$\frac{\Delta\Phi_1}{t'-0} = \frac{BA\left(\cos(0)-\cos\left(\frac{\pi}{3}\right)\right)}{t'} = \frac{BA\left(1-\frac{1}{2}\right)}{t'} = \frac{1}{2}\frac{BA}{t'}$$

$$\frac{\Delta\Phi_2}{2t'-t'} = \frac{BA\left(\cos(\pi/3)-\cos(0)\right)}{t'} = BA\left(\frac{1}{2}-1\right) = -\frac{1}{2}\frac{BA}{t'}$$

Plot



oscilates

angular speed of rotating loop

$$\varphi = \omega t$$

$$\boxed{\mathcal{E}(t) = BA\sin(\omega t)\,\omega}$$