Ohm's Law

What happens when we double V?

→ more and faster e^- movement

Double V → I doubles.

$I \propto V$

Lets look at the units though

$I \rightarrow \frac{C}{s}$ $\quad V \rightarrow \frac{J}{C}$ to get

$V = IX$ $\quad X$ must be $\frac{V}{A}$

? Recognize that the amount of current that can flow per Volt depends on the material.

classical picture to imagin resistance

for $V = \underbrace{I}\,X$

This should be large even for small V if the e^- can travel pretty freely

we call X → R for Resistance

$$\boxed{V = IR \rightarrow \text{Ohms Law}}$$

From above $R \rightarrow \frac{V}{A}$ ← called "ohm"

symbol for an Ohm is Ω (capital omega)

Wires in most electronics → copper

How much resistance does copper offer?

Depends on how much copper there is

 ← has less resistance than → 

Also depends on cross section.

In general:

$$R = \rho \frac{L}{A}$$

L ← Length of wire

A ← area of cross sec.

ρ ← rho, resistivity

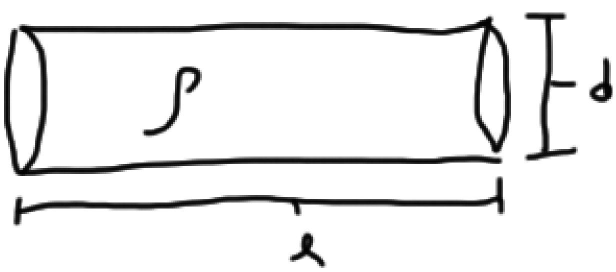
Know R unit is Ω

what are units of ρ? $\frac{L}{A} \to \frac{m}{m^2} = \frac{1}{m}$

so $\boxed{\rho \to \Omega \cdot m}$

Ex.

Given resistivity of copper what is the resistance of a round wire. You measure the diameter of the wire to be d and the length to be l

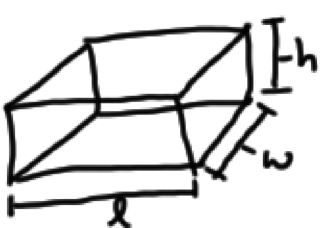


$$R = \rho \frac{L}{A} = \rho \frac{l}{\pi (d/2)^2}$$

$$\boxed{R = \rho \frac{l4}{\pi d^2}}$$

Ex 2

Building a circuit. Constrained to l & w but not h ussing copper (know ρ) what is h such that $R = R_{want}$



$$R = \rho \frac{L}{A} \qquad R_w = \rho \frac{l}{wh}$$

$$\boxed{h = \rho \frac{l}{w R_w}}$$

Compare some resistances

$\rho_{copper} = 1.7 \times 10^{-8}$
$\rho_{gold} = 2.44 \times 10^{-8}$
$\rho_{rubber} = 10^{13} - 10^{16}$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ look at wire
10cm long 1mm diam.

$$R_{copper} = \rho_c \frac{.1m}{\pi (1\times10^{-3}/2)^2} = \frac{1.7\times10^{-8} \times .1m \times 4}{\pi \; 10^{-6}} = \frac{1.7\times10^{-3}}{\pi}\times 4$$

$$\boxed{\approx 2\times10^{-3}\,\Omega}$$

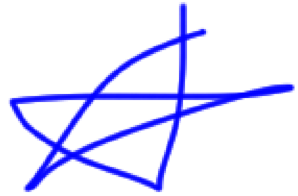
$$R_g = \rho_g \frac{.1m}{\pi (1\times10^{-3}/2)^2} = \boxed{3.1\times10^{-3}\,\Omega}$$

$R_r \approx 1\times10^{19}$

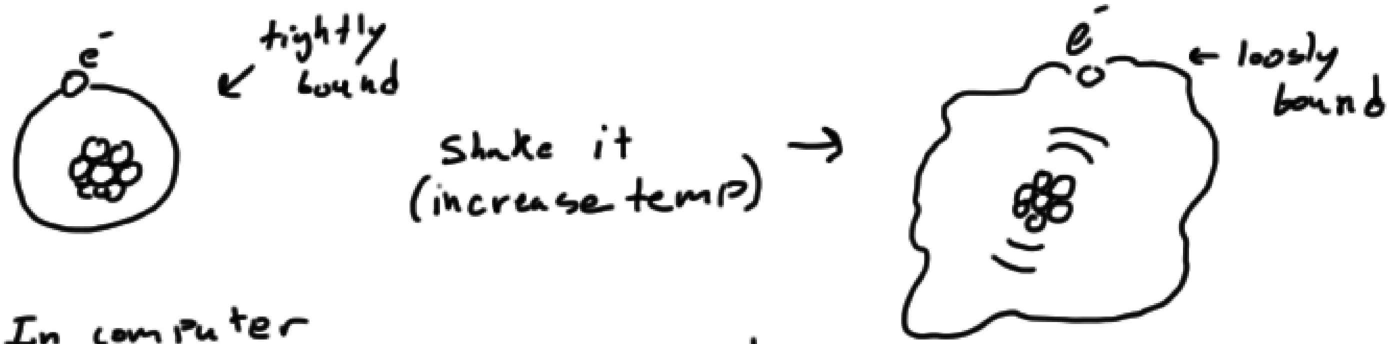
Different materials have different ρ.
How could we chang ρ of the same material.
If the atoms in a material are moving around a lot the will dirupt the movement of electrons.

→ Temperature measures how much the atoms are moving around

However increasing the temp. of a semiconductors increases the conductivity
↑
Very important concept



Add energy to a sem.cond.
electrons can more easily separate from atom → when seperate they can move from an applied potential.



In computer switches are made with sem.cond.s which are off (insulating) nomaly and can be turned on with an $\bar{E}$ field that loosens the electrons.

Power

Current in a circuit transfers energy. When charge moves in a circuit it moves from a high potential to a lower potential.

From 1st physics class we know Power (P):

$$P = \frac{\Delta \text{energy}}{\Delta \text{time}} \quad \text{in circuit} \rightarrow P = \frac{\Delta q V}{\Delta t}$$

I

So $\boxed{P = IV}$ units → W (watt)

In circuit erergy is given to device → to conserve energy then energy from battery is lost → does the potential across the battery change? It would but chem. reaction does work to keep potential const.

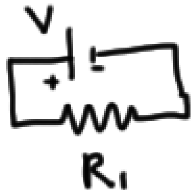
An electrical component using energy will have a resistance.

$\boxed{P = IV}$ $\quad V = IR \quad$ so $\quad P = I(IR) = \boxed{I^2 R}$

or $P = \frac{V}{R} V = \boxed{V^2/R}$

Series resistance

1 component



$V = IR$

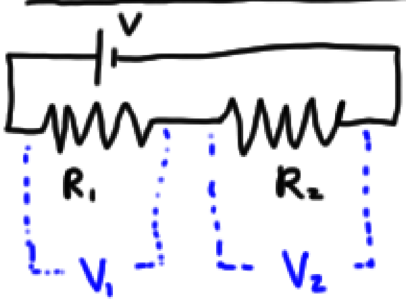
If I give you V & R_1
You can find I

$$I = \frac{V}{R_1}$$

Even without I you can find Power

$P = \frac{V^2}{R_1}$ or with I $P = I^2 R_1$

2 components in series



Same I through each
but different V across each

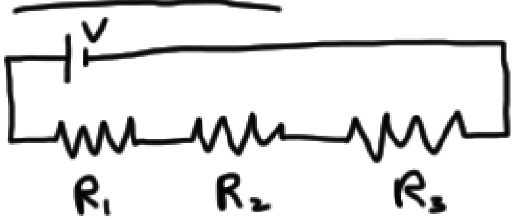
But total pot. across each must add to the total V.

so $V = V_1 + V_2 = IR_1 + IR_2 = I\underbrace{(R_1 + R_2)}_{} = IR_s$

$\boxed{\text{series resistance is just the sum of each}}$

$$R_s = \sum_n R_n$$

3 components



given: I, V, R_1, R_2

find: R_3

$R_s = R_1 + R_2 + R_3$ $\quad V = IR_s \quad V = I(R_1 + R_2 + R_3)$

$$\boxed{R_3 = \frac{V}{I} - R_1 - R_2}$$

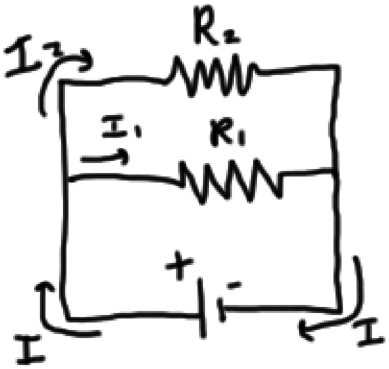
Power?

$$P = I^2 R_s = I^2\left(\cancel{R_1} + \cancel{R_2} + \frac{V}{I} - \cancel{R_1} - \cancel{R_2}\right)$$

$\boxed{P = IV}$ ← already knew that!

Parallel Ressistors

1) same votage accross components in parallel.
2) not <u>necessarily</u> the same current throu



Current must be conserved so $I = I_1 + I_2$

we know $I = \frac{V}{R}$

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

but statement 1) tells us $V_1 = V_2$

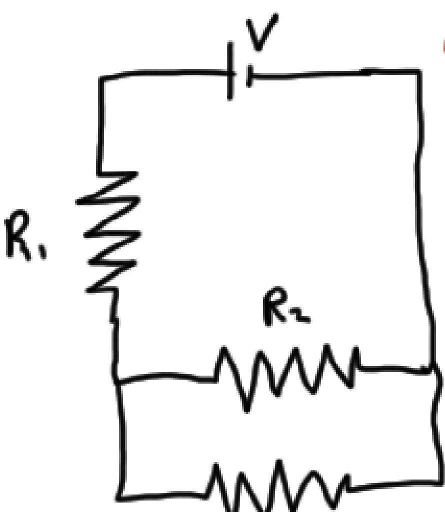
$$\text{So } I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad V = I\underbrace{\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right)}_{R_p}$$

$$\boxed{V = I R_p}$$

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Problems with both serries and parallel

$\boxed{\text{Total } R = ?}$



Simplify step by step

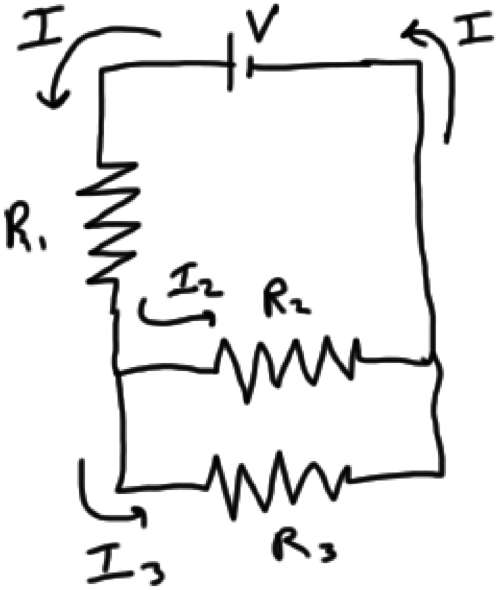
R_2 in parl. with R_3

$$\frac{1}{R_{13}} = \frac{1}{R_2} + \frac{1}{R_3}$$

R_1 in series with R_{13}

So R_{123} or $R_{tot} = R_1 + R_{13}$

$$\boxed{R_{tot} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}}$$



Give: V, I, R_1, R_2

find: R_3

If we knew V_3 and I_3 then $R_3 = \frac{V_3}{I_3}$

- R_3 & R_2 in parallel so $V_3 = V_2$
- $V_1 + V_{23} = V$
- $I = I_2 + I_3$

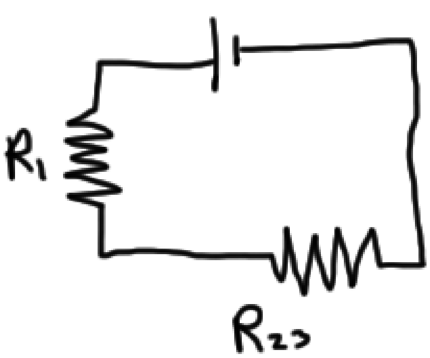
$$V_3 = V - V_1$$

$R_3 = \frac{V_3}{I_3}$ just need I_3

we could write $I_3 = I - I_2$

but we would still need I_2

→ Next step is to simplify the circuit a little



$\rightarrow V_1 + V_{23} = V$

$IR_1 + V_{23} = V$

$V_{23} = V - IR_1$

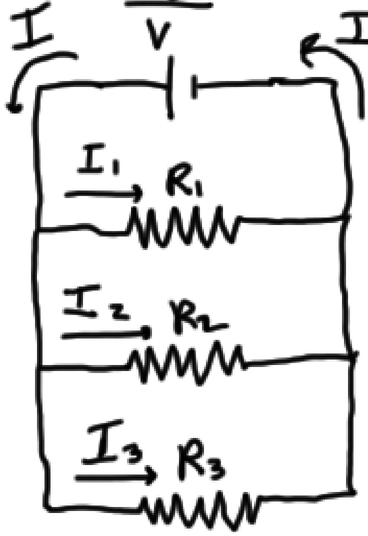
$V_3 = I_3 R_3 \rightarrow V - IR_1 = I_3 R_3$

$V_2 = I_2 R_2 \rightarrow V - IR_1 = I_2 R_2$

$I_2 = \frac{V - IR_1}{R_2}$ $\quad I_3 = I - \frac{V - IR_1}{R_2}$

$$R_3 = \frac{V - IR_1}{I - \frac{V - IR_1}{R_2}}$$

Ex



Find all I given V, R_1, R_2, R_3

all in parallel so:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

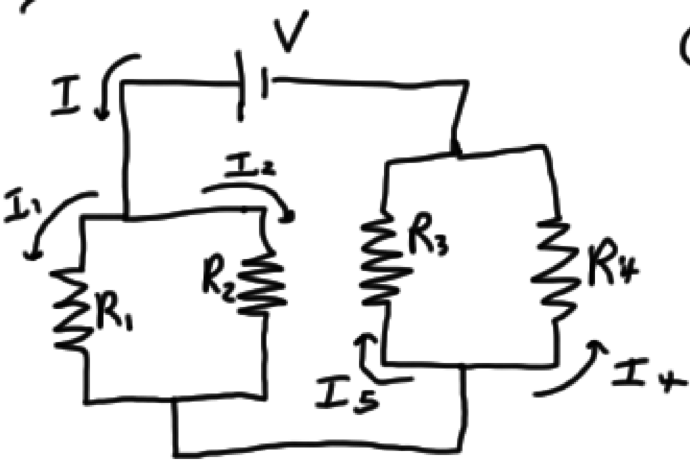
$V = I R_{tot}$ so $\boxed{I = \dfrac{V}{\left(\dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}\right)}}$

$$I = I_1 + I_2 + I_3$$

$$V_1 = V_2 = V_3$$

$$I_1 = \frac{V_1}{R_1} \qquad I_3 = \frac{V_3}{R_3} \qquad \boxed{I_n = \frac{V}{R_n}}$$

$$I_2 = \frac{V_1}{R_3}$$



Given V, R_1, R_2, R_3, R_4

find I_4

start with a relationship (eqn.) that contains what you know

$$V_4 = I_4 R_4$$

Don't know Don't know

start relating things we dont know to things we do know.

$V_4 = V_3$ $\qquad I = I_1 + I_2$

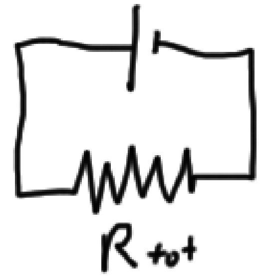
$V_1 = V_2$ $\qquad I = I_3 + I_4$

$V_{12} + V_{34} = V$ $\qquad V = I R_{tot}$

R_{tot}

1st simplify parallel resistors

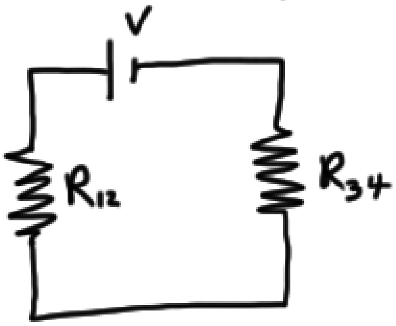
$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}$$



$$R_{tot} = R_{12} + R_{23} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} + \frac{1}{\frac{1}{R_3} + \frac{1}{R_4}}$$

$$I = \frac{V}{R_{tot}}$$

Unsimplify a little



$$V = V_{12} + V_{34}$$

$$V_{34} = V - V_{12}$$

$$V_{34} = \underbrace{V}_{\text{know}} - \underbrace{I}_{\text{know}} \underbrace{R_{12}}_{\text{know}}$$

Back to

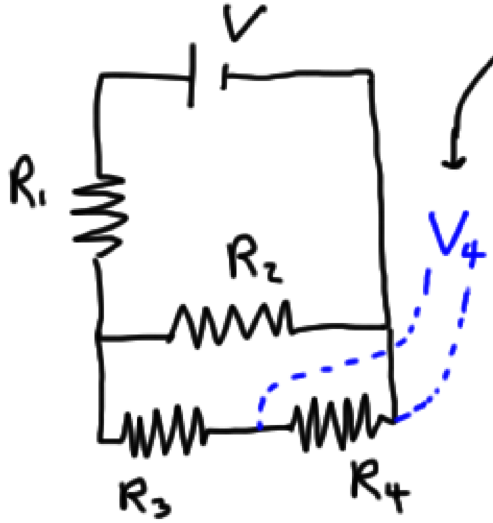
$$V_4 = I_4 R_4 \qquad V_4 = V_3$$

so $I_4 R_4 = V - I R_{13}$

$$\boxed{I_4 = \frac{V - IR_3}{R_4}}$$

Might include full expression of I from above

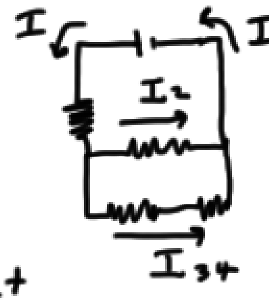
Another resistor problem



What is V_4? i.e. what is the potential across R_4?

Given: V, R_1, R_2, R_3, R_4

Draw in your current

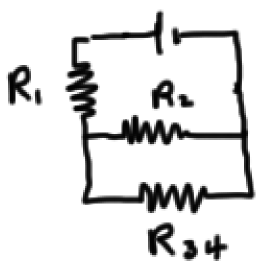


Write an expression that contains what we want to know

$V_4 = I_{34} R_4$

Don't know

simplify ciruit.

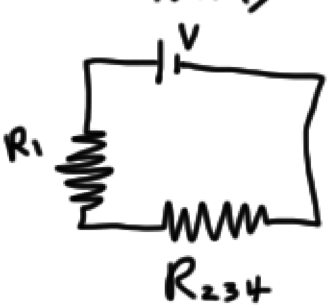


$V_{34} = V_2$ $\qquad I = I_2 + I_{34}$

$V_3 + V_4 = V_{34}$

$$\frac{I_{34}}{R_3} + \frac{I_{34}}{R_4} = V_{34} \quad (1)$$

Simplify again



$V_2 = V_{34} = I R_{234}$

$V_2 = I_2 R_2$

$$I_{34} = I - I_2 = \frac{V}{R_{tot}} - \frac{V_2}{R_2} = \frac{V}{R_{tot}} - \frac{V_{34}}{R_2}$$

$$(1) \rightarrow V_{34} = \frac{V}{R_t R_3} - \frac{V_{34}}{R_2 R_3} + \frac{V}{R_t R_4} - \frac{V_{34}}{R_2 R_4}$$

$$V_{34} + \frac{V_{34}}{R_+ R_3} + \frac{V_{34}}{R_2 R_4} = \frac{V}{R_+}\left(\frac{1}{R_1} \cdot \frac{}{R}\right.$$

$$V_{34}\left(1 + \frac{1}{R_+R_3} + \frac{1}{R_2R_4}\right) = \frac{V}{R_+}\left(\quad 3 + \right.$$

$$V_{34} = \frac{V(R_4+R_3)}{R_+R_3R_4\left(1+\frac{1}{R_+R_3}+\frac{1}{R_2R_4}\right)}$$

$$\boxed{V_{34} = \frac{V(R_4+R_3)}{R_+R_3R_4 + R_4 + R_+R_3/R_2}}$$

Skip algeb in class.

we have the idea

Kirchoff's Rules

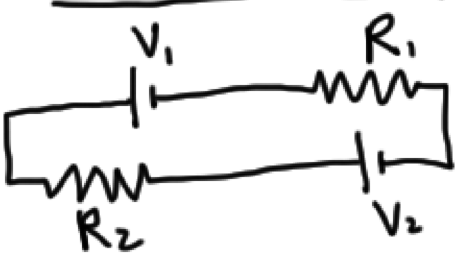
Junction Rule → already been using

Current in = current out

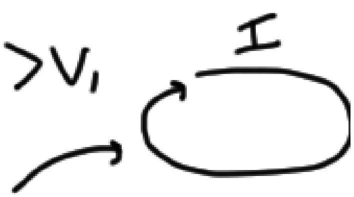
Loop rule

In a given loop the potential drop = Potential gain

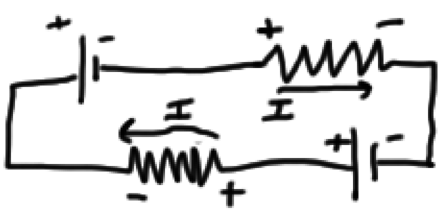
Ex of loop rule



for $V_2 > V_1$



If you choose this incorrectly it will work out . e (get a different sign)

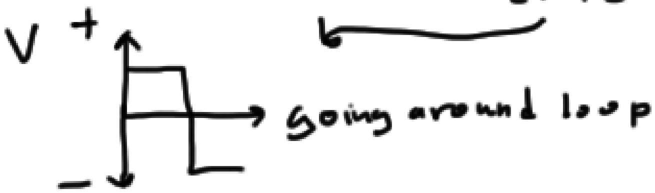


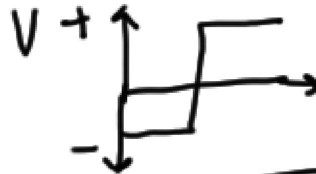
going clockwis a potential drop is ⊕ → ⊖ (Resistor).

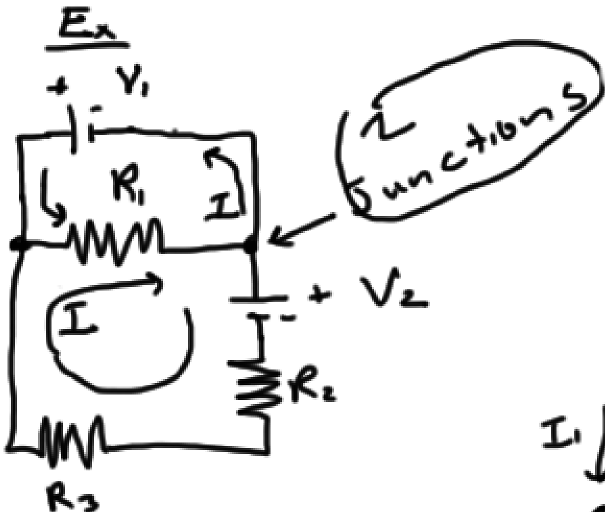
Potential gain ⊖ → ⊕.

$V = IR$ so

$$\underbrace{V_1 + IR_1 + IR_2}_{\text{drops}} = \underbrace{V_2}_{\text{gains}}$$

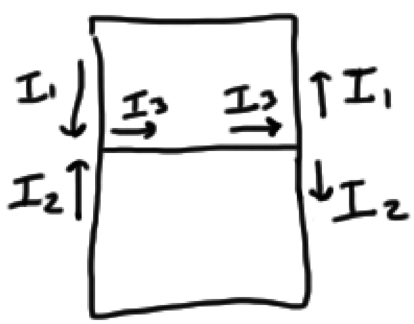






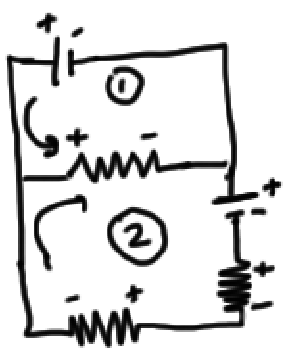
1st Junction rule

redraw current in and out of junctions



so $\boxed{I_2 + I_1 = I_3}$

Now loop rule



loop ① follow current counterclockwise from uper left corner

$$\boxed{I_3 R_1 = V_1}$$

loop ② clockwise uper left corner

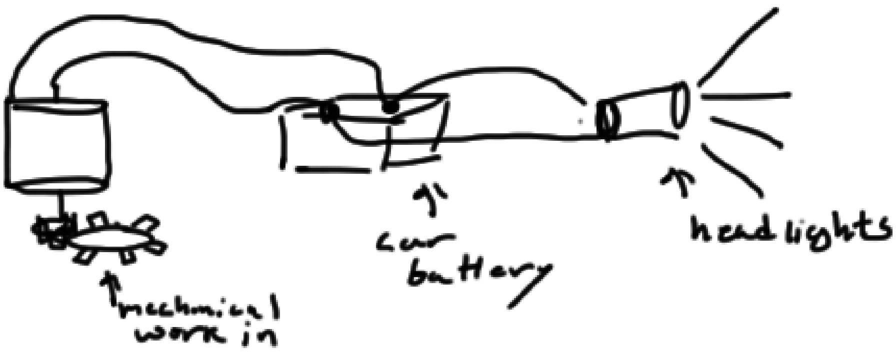
$$\boxed{I_3 R_1 + V_2 + I_2 R_2 + I_2 R_3 = 0}$$

If you were given R_1, R_2, R_3, V_1, V_2 could you find all I (I_1, I_2, I_3)?

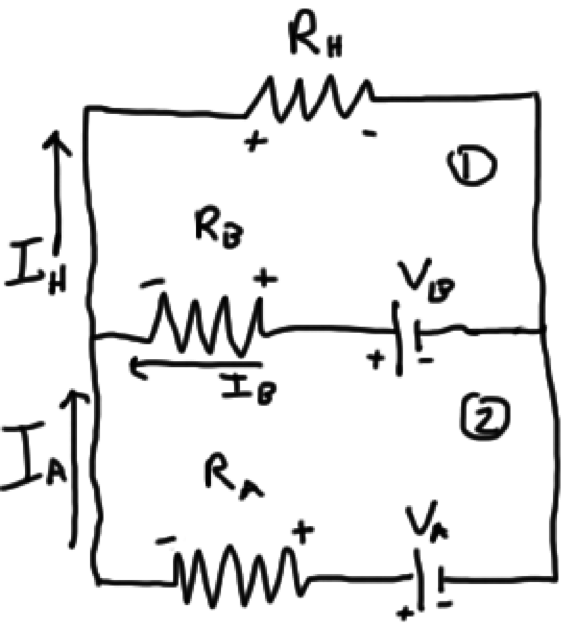
Yes → 3 unique equations and 3 unknowns

Batteries can have internal resistance.
Such a battery could be modeled like

<u>Ex. from book</u> (no #s here)



Both the battery and alternator have internal resistance
Circuit:



Junction rule: $I_B + I_A = I_H$

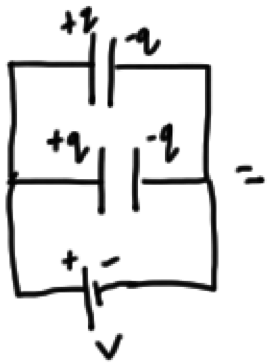
loop rule:

① $\underbrace{I_H R_H + I_B R_B}_{\text{drop}} = \underbrace{V_B}_{\text{gain}}$

② $\underbrace{V_B + I_A R_A}_{\text{drop}} = \underbrace{V_A + I_B R_B}_{\text{gain}}$

given $V_A, V_B, R_A, R_B, R_H \rightarrow$ can find all I
(3 eqns 3 unknowns)

Capacitors in Series and Parallel



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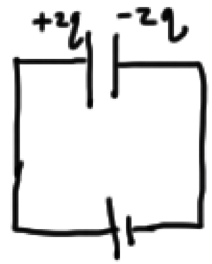
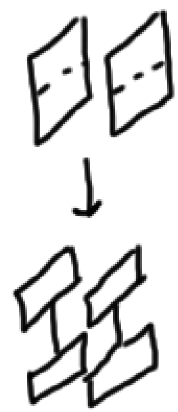


→ like



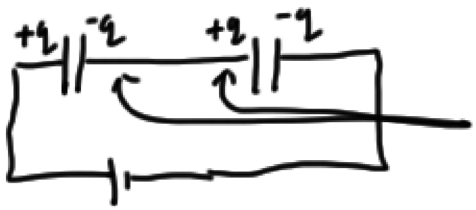
If you Don't change A or d you keep the capacitance the same →

$$\boxed{q = CV}$$

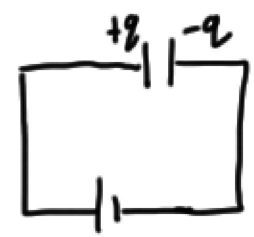
So capacitors is parallel:

$$\boxed{C_p = C_1 + C_2 + \dots}$$

What abount in series



+q & -q cancel →



$$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} = q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \qquad q = \frac{V}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \dots}$$

Opposite to resistors when $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$ and $R_s = R_1 + R_2 + R_3$.

Using these addition rules & $q = CV$ You can solve for various quantities in much the same way as the resistor problems.