I. ELECTRIC CHARGES FORCES AND FIELDS

The atom

- Protons
- Neutrons
- Electrons

The classical picture of an atom is shown in Fig. 1 but this is not really what and atom looks like. A hydrogen atom might "look" something more like whats shown in Fig. 2.

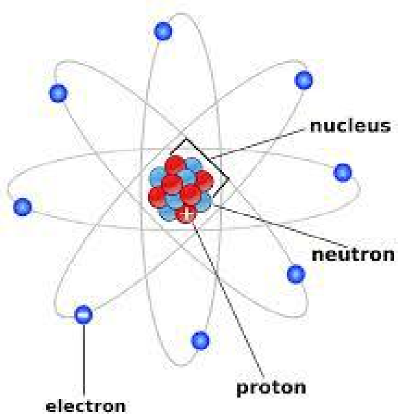


FIG. 1. A

Hopefully we can talk about why an atom looks something like this when we discuss quantum mechanics.

For now The classical picture of atom will be fine since charge comes in distinct unbreakable lumps. We can say that charge is quantized. Even though a real atom is a fuzzy strange object that can be difficult to describe this (Fig. 1) classical picture is helpful in remembering the constituents of an atom.

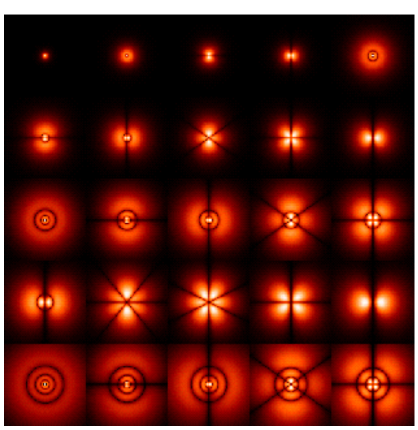


FIG. 2. caption

Lets define a couple of important terms.

- Z is often called the atomic number. This tells you how many protons are in the nucleus
- *Ion*: An atom with a different number of electrons than the atomic number. an Ion has a non-neutral charge.

If charge is Quantized and the electron and proton carry the smallest amounts of charge that something can have then how much charge is that?

$$e = 1.60 \times 10^{-19} C \tag{1}$$

The symbol e is the amount of charge on the electron and proton. Electron carries $-e$ And the proton carries e. C is the international system of units for charge called "Coulombs".

In nature the smallest charge we ever find on an object is e and any charge on any object can be written in terms of $N \times e$. However e is so small that amount of charge on a daily object can be treated as continuous.

So how many electrons are there in 1 C?

$$\frac{1.00C}{1.60 \times 10^{-19} C} = ? \tag{2}$$

The coulombs cancel $\rightarrow$ a number of *something* is dimensionless, and we get

$$\frac{1.00C}{1.60 \times 10^{-19} C} = 6.25 \times 10^{18} = \tag{3}$$

Many!

e Is so small that it wasn't until 1909 that Robert A. Millikan and Harvey Fletcher Were able to measure the rough charge of an electron using what was called to Millikan oil drop experiment.

II. COULOMB DEMO

III. TRIBO CHARGING

In the books they mention Tribo Charging. This is giving an electrical charge to an object by rubbing against another object. This is actually a very complicated process. Rubbing two things together can give one a positive charge in one a negative charge. One becomes positive and the other becomes negative → charge must be conserved.

IV. INTERACTING CHARGES

How do two charges interact? Interact through the electric field. Ultimately though the interactions are performed by a different particle, the photon. A photon is the smallest quantum of light. It may be hard to imagine but all electric interactions are mediated by light.

- For our purposes it'll be sufficient to talk about the interaction through the electric field.
- Like charges repel. Opposite charges attract.
- For point particles this attraction or repulsion can be described with Coulomb's law.

Coulomb's Law:

Eg. Let's take two-point charges q_1 and q_2 separated by a distance r. The magnitude of the force F between the two charges is

$$F = k\frac{|q_1||q_2|}{r^2} \tag{4}$$

$k \rightarrow$ just a constant. The purpose of k is to scale the force by something so that our units have physical meaning. k itself has meaning, can also be expressed as

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \mathrm{Nm}^2 \tag{5}$$

ϵ_0 is an important number called the permittivity of free space. Related to the speed of light traveling through vacuum. Eureka!! The speed of light shows up in Coulomb's law!

Compaire with gravitational force

Take two protons, each with charge e And separate them by $1m$. What is the magnitude of the force between them?

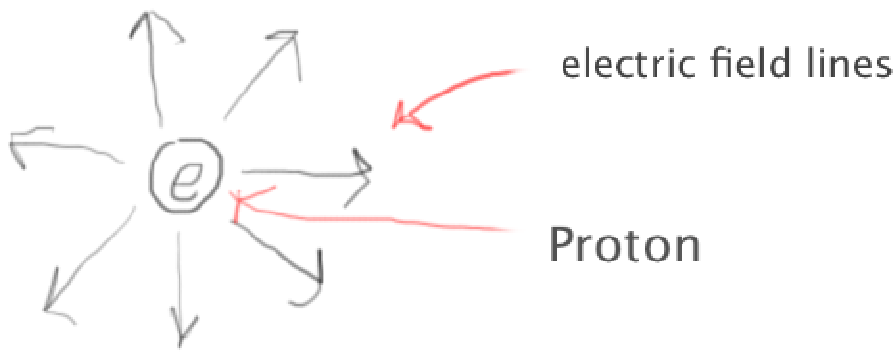
$$F_e = k\frac{|q_1||q_2|}{r^2} = 2.30 \times 10^{-28} \tag{6}$$

Now what about the gravitational force between these Same protons the same distance apart?

$$F_g = G\frac{m_1 m_2}{r^2} = 5.53 \times 10^{-71} \tag{7}$$

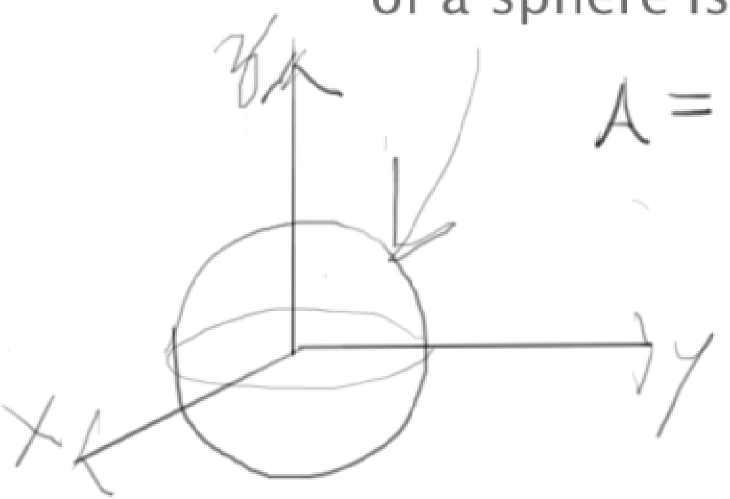
Wow! That is much smaller than the electric force between them. Another distinction is that the gravitational forces always attractive. The electric force can change sign depending on the charges that are interacting.

How can we understand the position dependence in Coulomb's law?



the surface area of a sphere is

$$A = 4\pi r^2$$



If the field lines communicate the force between two particles than how does the number of field lines depend on the distance between two particles?

Field lines per unit area?

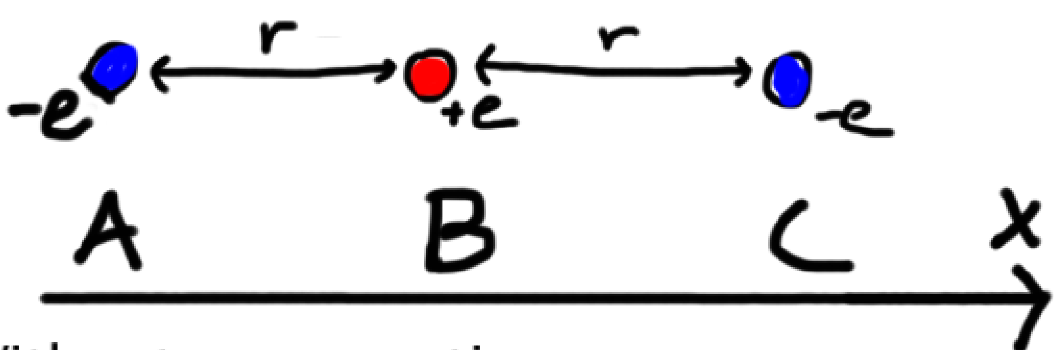
$N \rightarrow$ number of field lines in A

So the number of field lines per unit area is

$$\frac{N}{A} = \frac{N}{4\pi r^2} \propto \boxed{\frac{1}{r^2}}$$

Just like Coulomb's law

What happens when there are more than two charges?



Without any equations what is the net force on particle B?

$\rightarrow 0$

What about the net force on particle A?
More complicated but we only need to add the pairwise forces.

The force between two particles has a direction. Use vectors.

Force on A from B

$$\bar{F}_{AB} = \hat{r}_{AB} \, k \frac{|q_A||q_B|}{r_{AB}^2}$$

$\hat{r}_{AB} \rightarrow$ unit vector from A to B

BE CAREFUL WITH YOUR SINGNS !

No y component

so: $\bar{F}_{AB} = \frac{ke^2}{r^2}\hat{x} \leftarrow$ +x direction

What about $\bar{F}_{AC}$?

$$\bar{F}_{AC} = \frac{-ke^2}{(2r)^2}\hat{x}$$

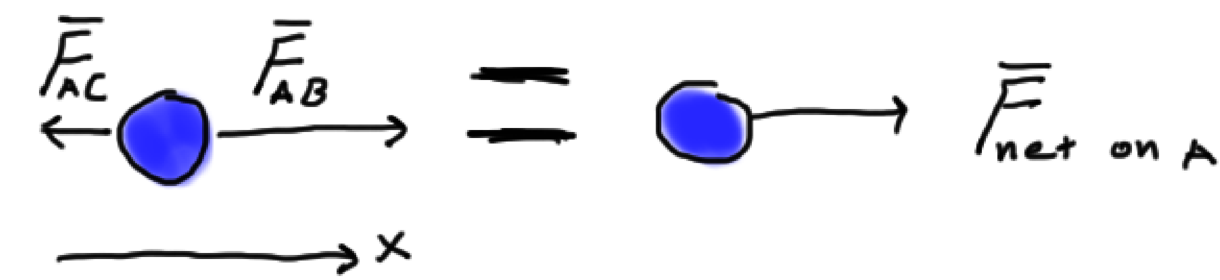
To get the total force on A we can just add the force from B & C.

$$\bar{F}_{\text{net on A}} = \bar{F}_{AB} + \bar{F}_{AC}$$

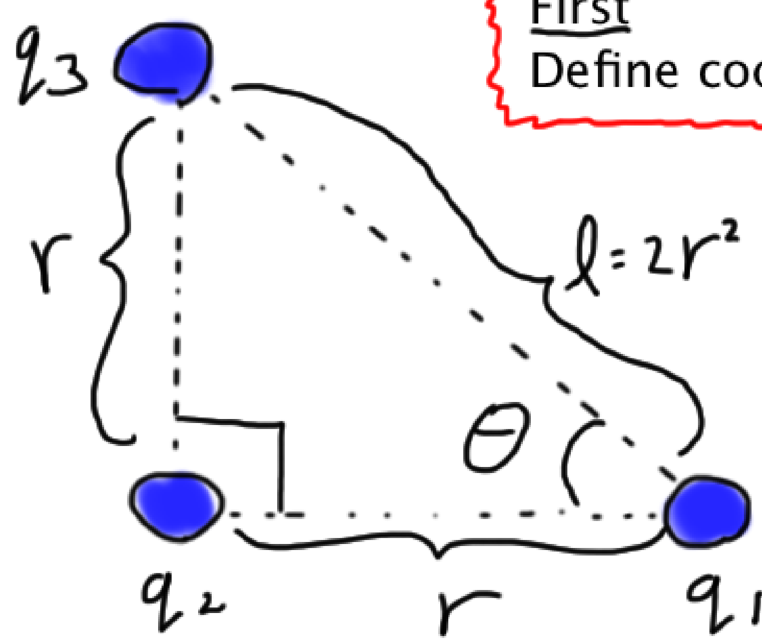
$$\bar{F}_{net\ on\ A} = \left(\frac{ke^2}{r^2} - \frac{ke^2}{(2r)^2}\right)\hat{x}$$

$$= \frac{\hat{x}k}{4r^2}\left(4e^2 - e^2\right) = \boxed{\frac{3ke^2}{4r^2}}$$

Free body diagram of A

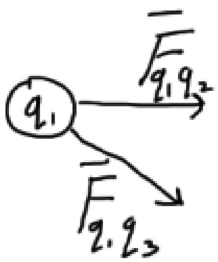


Lets try a problem in 2 dimensions.



$\boxed{\begin{array}{l}\underline{\text{First}} \\ \text{Define coordinate axes.}\end{array}}$

The question:
What is the force on q_1
from q_2 & q_3



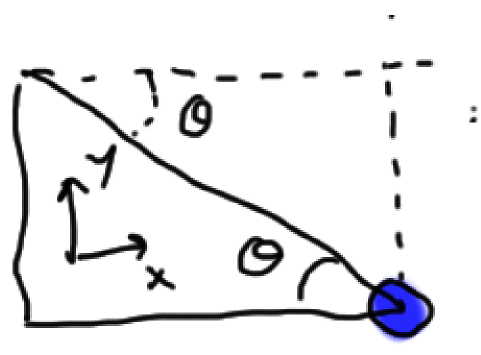
$\longleftarrow$ same notation as before

Split vectors into x and y components

$$\bar{F}_{q_1 q_2} = \left(F^{x}_{q_1 q_2} , F^{y}_{q_1 q_2} \right)$$

Other notation $\longrightarrow$ $\bar{F}_{q_1 q_2} = F^{x}_{q_1 q_2}\, \hat{x} + F^{y}_{q_1 q_2}\, \hat{y}$

$$\begin{cases} F^{y}_{q_1 q_2} = 0 \\ F^{x}_{q_1 q_2} = \dfrac{k|q_1||q_2|}{r^2} \end{cases}$$



$$\begin{cases} F^{y}_{q_1 q_3} = \dfrac{-k|q_1||q_3|}{l^2} \sin\theta \\ F^{x}_{q_1 q_3} = \dfrac{k|q_1||q_3|}{l^2} \cos\theta \end{cases}$$

$l^2 = 2r^2$

SO

$$\bar{F}_{net} = \left(\frac{kq_1}{r^2}\left[q_2 + \frac{q_3}{2}\cos\theta\right], \frac{-kq_1q_3}{2r^2}\sin\theta \right)$$

$$= \boxed{\frac{kq_1}{r^2}\left(q_2 + \frac{q_3}{2}\cos\theta, \frac{q_3}{2}\sin\theta \right)}$$

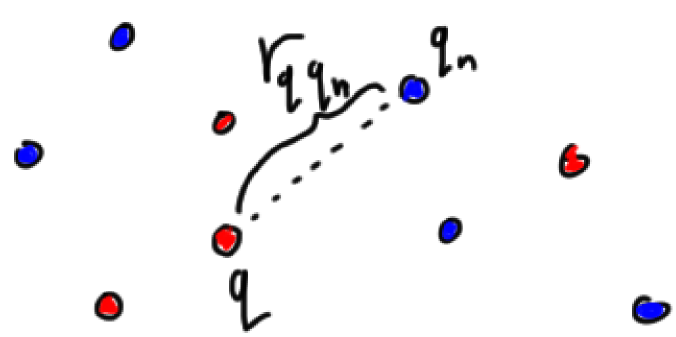
Lets generalize the force felt by a single particle from N other particles

Force on q from q_n

$$\boxed{\begin{aligned} \bar{F}_{net\ on\ q} &= \sum_{n=1}^{N} \bar{F}_{qq_n} \\ \bar{F}_{qq_n} &= \hat{r}_{qq_n} \frac{k|q||q_n|}{r_{qq_n}} \end{aligned}}$$

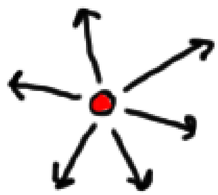
absolute distance between q & q_n

direction from q to q_n → don't forget sign from charges

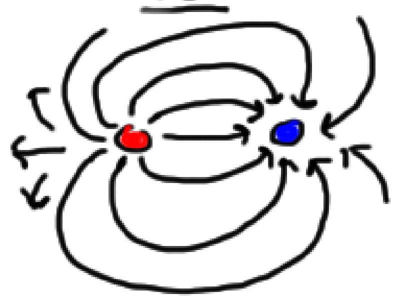


The Electric Field

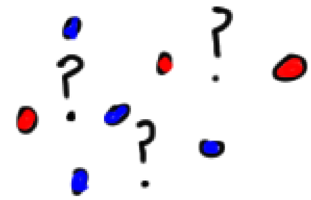
one particle



two



many



But what do these pictures mean?

$\bar{E}$ gives direction & mag. of force at every point in space.

$q_0 \rightarrow$ Positive test charge

$\rightarrow$ very small so it does not change the system around it

$$\boxed{\bar{E} = \frac{\bar{F}}{q_0}}$$

Force on test charge

Eg.

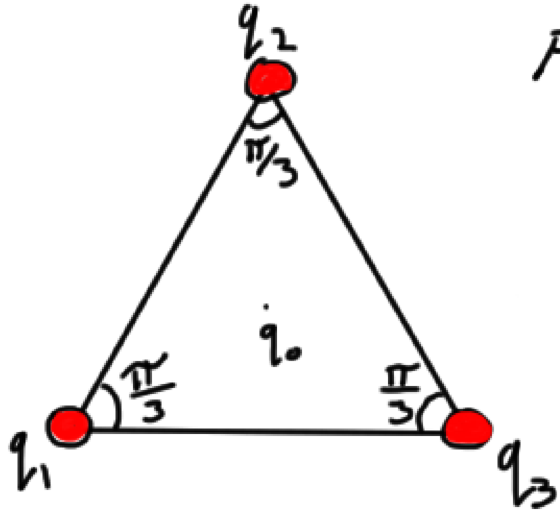
What is $\bar{E}$ from single particle q_1

we know $\bar{F} = \frac{k|q_1||q_0|}{r^2}$ so:

$$\bar{E}_{q_1} = \frac{k|q_1||q_0|}{q_0 r^2} = \boxed{\frac{k|q_1|}{r^2}}$$

Eg

What is net $\bar{E}$ at center?

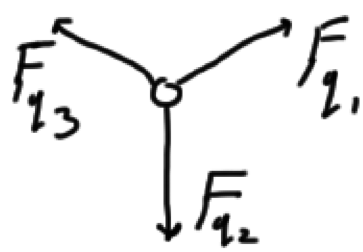


For $q_1 = q_2 = q_3$

1) Place test charg q_0 at center

2) write force on q_0

$$\bar{F}_{q_0} = \bar{F}_{q_0 q_1} + \bar{F}_{q_0 q_2} + \bar{F}_{q_0 q_3}$$

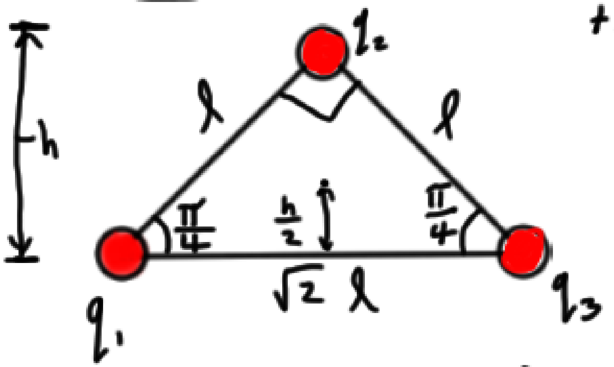


Sum to 0

So $\boxed{\bar{E}_{center} = 0}$

Eg

What about at center of this triangle?



$\bar{F}_{q_0} = ?$ $\Rightarrow$ From Symm. no horz. component

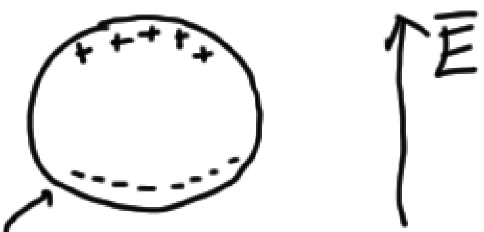
$$h = \frac{\sqrt{2}}{2} l$$

Conductors & Insulators

Charge can move easily through → conductors
can not move easily through → Insulators

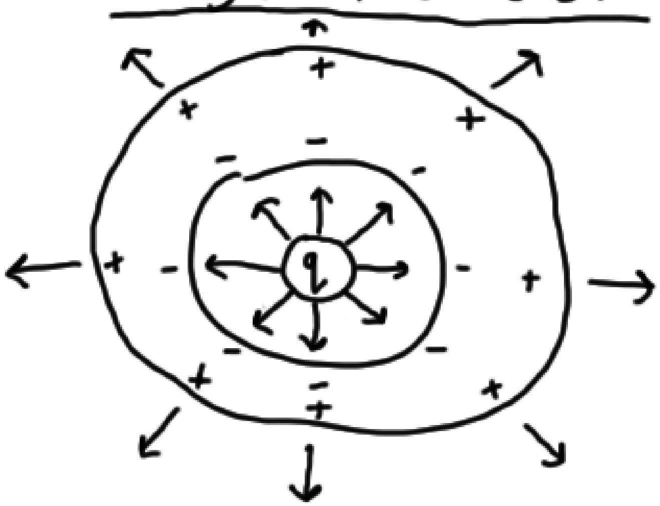
In materials charge carriers are electrons
In conductor imagine a mostly
free gas of electrons.

Conductor in $\bar{E}$ field?



electrons move
leaving (+) charges at the top

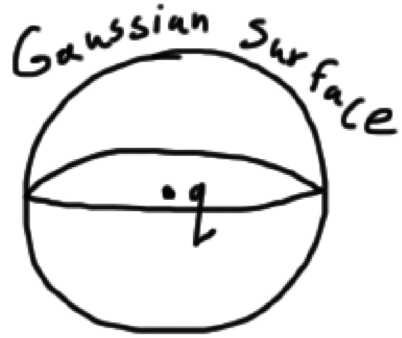
Charge in conductor?



Rules for drawing field lines in/out from conductors
1) Perpendicular at surface
2) $\bar{E}_{inside} = 0$

Gauss' Law

Relates charge distribution
to electric field.



single particle

$|E| = \frac{kq}{r^2}$ $\quad k = \frac{1}{4\pi\epsilon_0}$

$|E| = \frac{q}{4\pi\epsilon_0 r^2}$ $\quad A_{sphere} = 4\pi r^2$

$|E| = \frac{q}{A\epsilon_0} \rightarrow \boxed{A|E| = \frac{q}{\epsilon_0}}$

Gauss' law
for point charge in sphere

$\rightarrow$ NO r Dependence

$|E|A \rightarrow$ simple example of <u>electric flux</u>

Symbol for elec. flux $\rightarrow \Phi_E$

More Generally

$$\Phi_{E \text{ over } \Delta A} = \Delta A |E| \cos\varphi$$

$\Phi_{E \text{ for } \Delta A} = \Delta A\, \bar{E} \cdot \hat{n}$

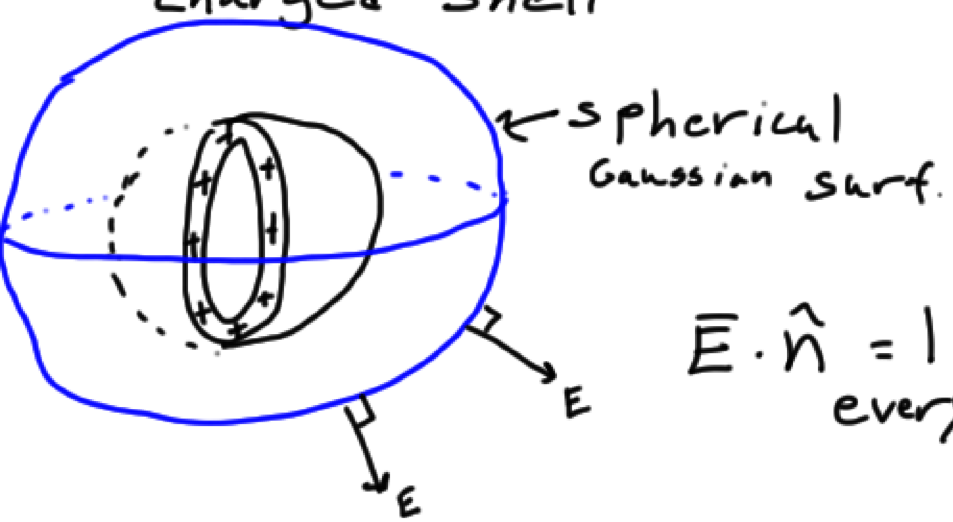
$$= \Delta A\, |E| \cos\varphi$$

Total flux

$$\boxed{\Phi_E = \sum \Delta A\, E \cos\varphi = \frac{Q}{\epsilon_0}}$$

Important Ex.

Spherical charged shell

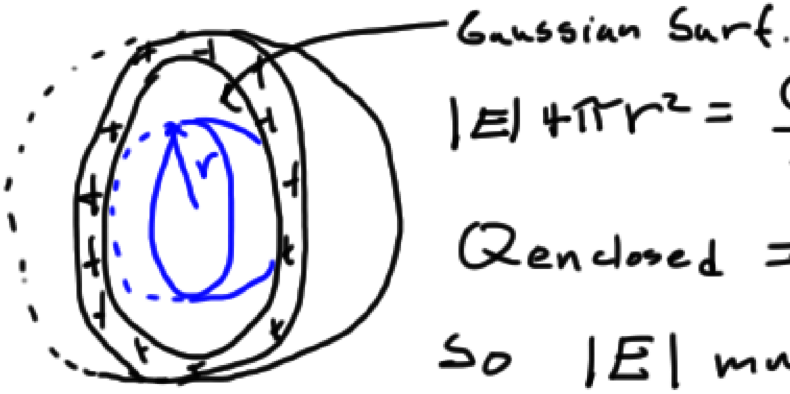


$E \cdot \hat{n} = |E|$ everywhere

So $\Phi_E = A|E| = \frac{Q}{\epsilon_0} \rightarrow 4\pi r^2 |E| = \frac{Q}{\epsilon_0}$

good → same as $|E| = \frac{Q}{4\pi r^2 \epsilon_0}$ ← Point Particle

What about:

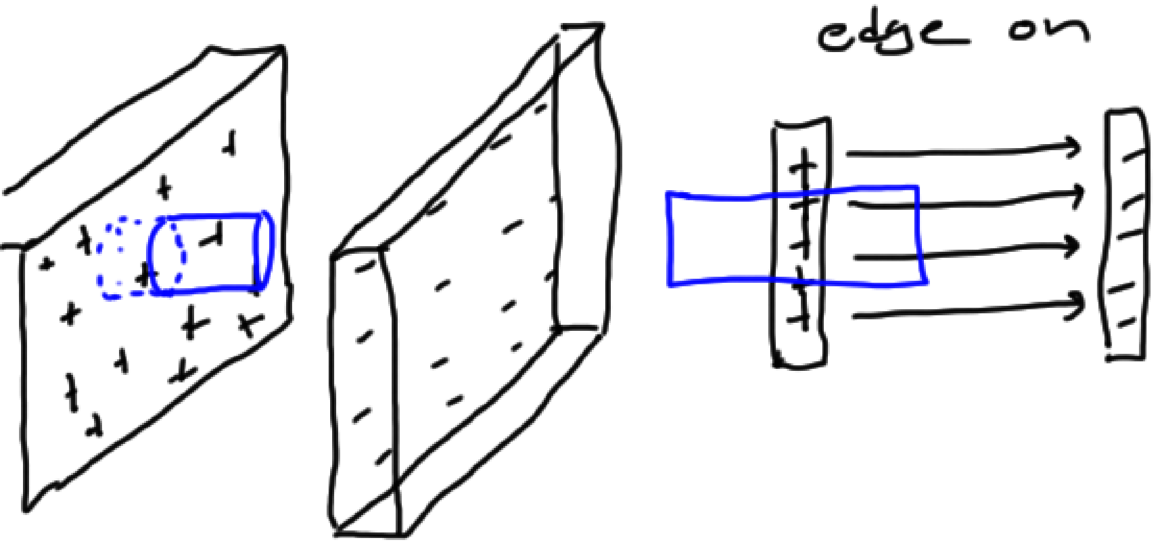


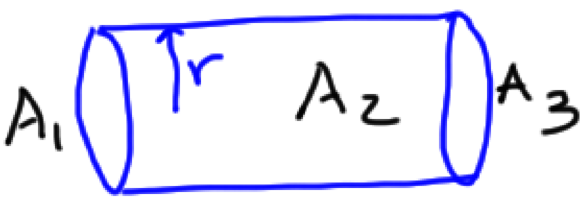
$$|E|\,4\pi r^2 = \frac{Q}{\epsilon_0}$$

$Q_{enclosed} = 0$

so $|E|$ must $= 0$!

What is $|E|$ and charge dist. for:





$\Phi_{EA_1} = 0$ (in conductor)

$\Phi_{EA_2} = 0$ $(\bar{E}\cdot\hat{n} = 0)$

$\bar{\Phi}_{EA_2} = \bar{E}\cdot\hat{n}\,A_2 = |E|\pi r^2$

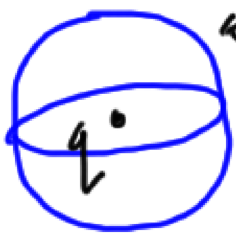
so $|E|\pi r^2 = \dfrac{Q}{\epsilon_0}$ $\quad Q = \sigma \pi r^2$ ← charge per unit area

Now we can write:

$$|E|\cancel{\pi}\cancel{r^2} = \frac{\sigma\cancel{\pi}\cancel{r^2}}{\epsilon_0}$$

and $\boxed{|E| = \frac{\sigma}{\epsilon_0}}$

We know



for Gauss. surf

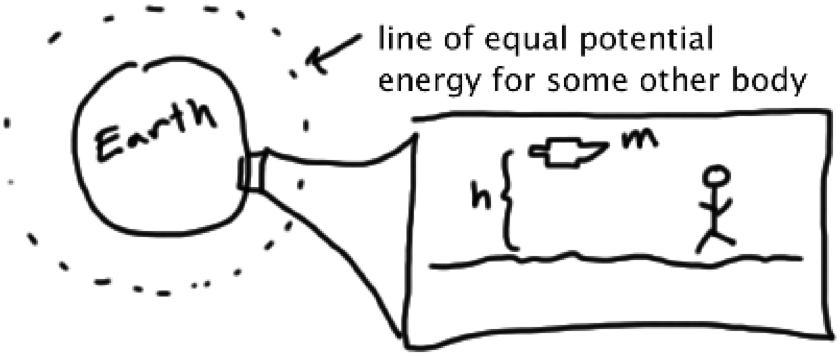
$$|E|A_s = \frac{q}{\epsilon_0}$$

What about:

even out to

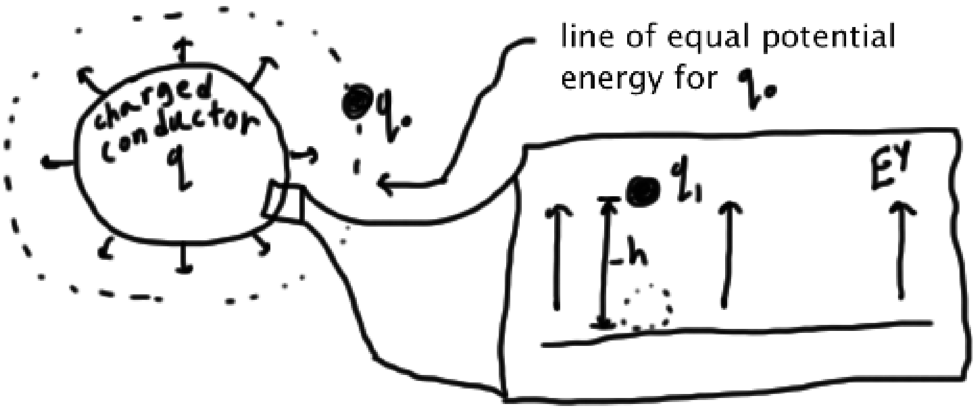
$$|E|A_s = \frac{q}{\epsilon_0}$$

Electric potential



Close to the surface the gravitational field is uniform. $\Big\} \rightarrow PE = mgh$

Very similar story for electric potential



Close to the surface the electric ~~gravitational~~ field is uniform.

$$\boxed{PE = E^y q_0 h}$$

(choose surface as zero potential)

Book calls EPE

can think about in terms of work

$$\bar{F}_{\text{on } q_0} = q_0 \bar{E} \qquad W = \bar{F} \cdot \bar{D} = q_0 \bar{E} \cdot \bar{D}$$

$$W_{AB} = EPE_A - EPE_B$$

$$V \equiv \frac{EPE}{q_0} \leftarrow \text{Definition}$$

Electric Potential

$$\boxed{\Delta V = \frac{-W_{AB}}{q_0}}$$

Electric Potential Difference from Point Charge

Must use calculus so we will
have to take it as a rule.

$$\boxed{V = \frac{kq}{r}}$$

Intuitively → sort of like

$$\frac{\bar{F} \cdot \bar{D}}{q_0}$$

say $\bar{D}$ along $\hat{r}$

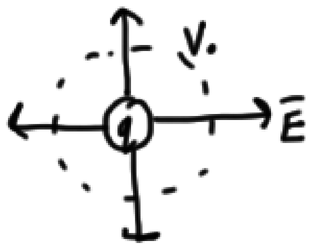
Since $\bar{F} \cdot \bar{D} = \hat{r}\,\frac{k q_0 q}{r^2}\,\bar{D}$

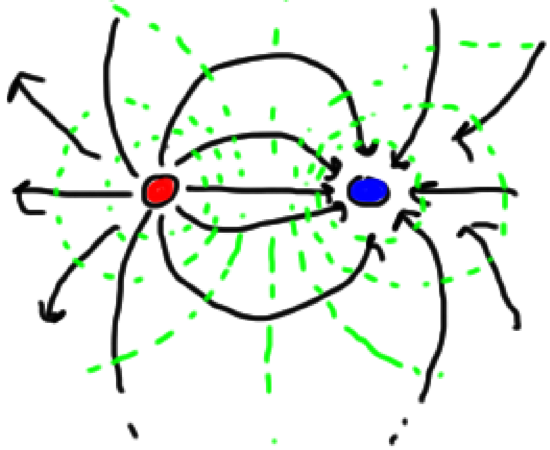
then $\frac{\bar{F} \cdot \bar{D}}{q_0} = \hat{r}\,\frac{\cancel{q_0} q}{\cancel{q_0} r^{\cancel{2}}}\,\cancel{r} = \hat{r}\,\frac{q}{r}$

Not exact but hopefully clarifying.

Equipotential Surfaces

→ Always perpendicular to electric field lines





→ Net force does no work along equipotential

→ Equipotential line in 2D.

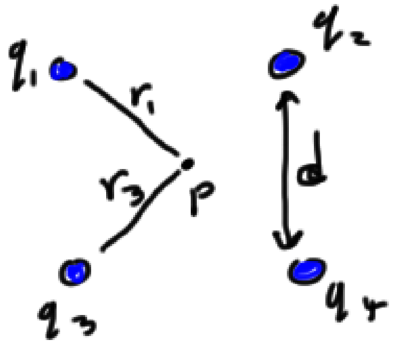
→ Equipotential surface in 3D.

Can sum over potential from
individual bodies to get total potential.

$$V_{tot}(P) = \sum_n V_n(P)$$

at point P

Ex.



$q_1 = q_2 = q_3 = q_4 = -q$

← square

What is $V_{tot}(P)$

$$V_{tot}(P) = \sum_n \frac{kq_n}{r_n}$$

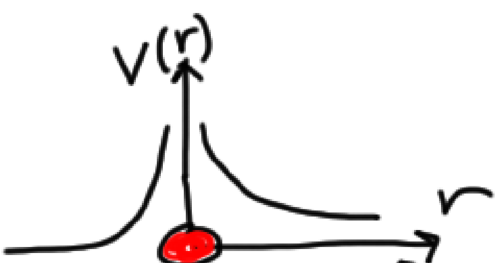
$r_1 = r_2 = r_3 = r_4$ → since P is equidistant from all q_n.

We know $\sqrt{2d^2}$ = Square diag.

$$\rightarrow \boxed{r = \frac{\sqrt{2}}{2} d}$$

Now: $V_{tot}(P) = \frac{-kq}{r} - \frac{kq}{r} - \frac{kq}{r} = \boxed{-\frac{6kq}{\sqrt{2}d}}$

What is V at $r=0$?

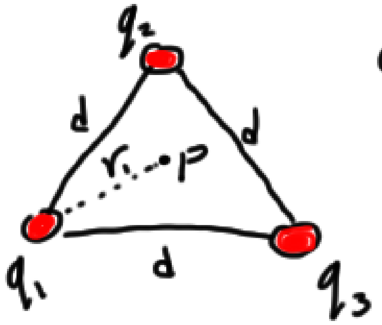


$V(r=0) = \infty$

What about $V(r=\infty)$?

$V(r=\infty) = 0$

In class ask students to find V(P) for



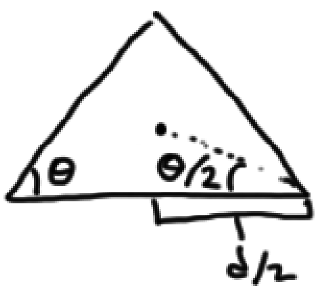
$$q_1 = q_2 = q_3 = q$$

$$r_1 = r_2 = r_3$$

$$V_{tot}(P) = \frac{kq}{r} + \frac{kq}{r} + \frac{kq}{3}$$

$$V_{tot}(P) = \frac{3kq}{r}$$ ← Just need r

for equilat. triang.

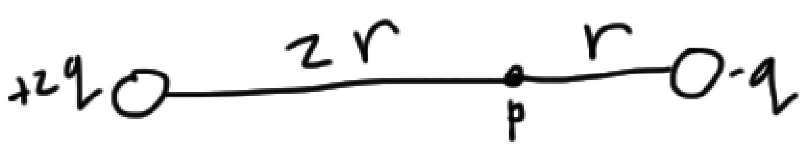


$$\theta = \pi/3$$

$$\cos(\theta/2) = \frac{d/2}{r}$$

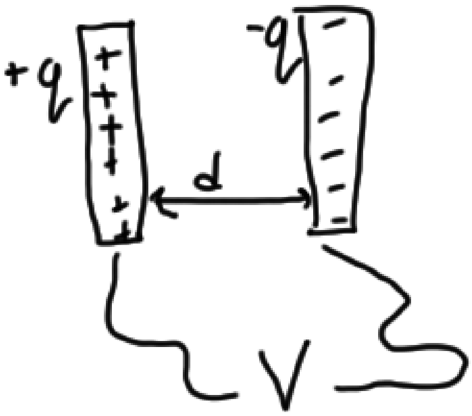
$$r = \frac{d}{2\cos(\theta/2)} = \frac{d}{2\sqrt{3}/2} = \frac{d}{\sqrt{3}} = \boxed{\frac{\sqrt{3}\,d}{3}}$$

$$V_{tot}(P) = \frac{3kq}{\sqrt{3}\,d/3} = \boxed{\frac{9kq}{\sqrt{3}\,d} = V_{tot}(P)}$$



$$V_t(P) = \frac{\cancel{2}kq}{\cancel{2}r} - \frac{kq}{r} = 0$$

Capacitors



$q = CV$ ← electric potential

↑ capacitance

C → measured in units of farad

farad $(F) = \frac{\text{coulomb}}{\text{volt}}$ ← SI units

Can apply potential V to charge the plates. The E inside is then related by

$|E| \rightarrow E = \frac{V}{d}$

How can we increase the C?

$C = \frac{q}{V} = \frac{q}{dE}$ $\quad E = \sigma / \epsilon_0$

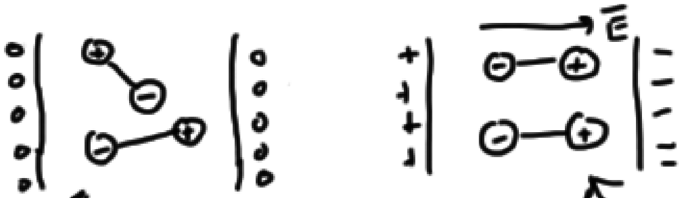
$\sigma = \frac{q}{A}$ ← area of plate

$C = \frac{q}{d \frac{q}{A\epsilon_0}} = \boxed{\frac{A\epsilon_0}{d} = C}$

C big for small d & large A

How else could we increase C?

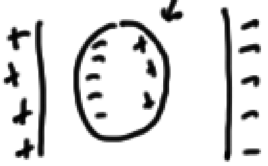
→ Add Dielectric



work done to align

dipolar molecules can align with $\bar{E}$

or



work done to separate

These fillings increase C

Call K dielectric const.

$$K = \frac{E_0}{E}$$

E_0 ← size of electric field with nothing inside

E ← " " dielectric inside

For a good dielectric $E \to$ small

so good dielectric has large K

what is expression for C?

$$E = \frac{E_0}{K} = \frac{V}{d} \qquad \frac{\sigma}{\epsilon_0} = E_0 \;\to\; \frac{q}{A\epsilon_0} = E_0$$

$$E_0 = \frac{KV}{d} \quad \text{so} \quad \frac{KV}{d} = \frac{q}{A\epsilon_0} \qquad q = \frac{A\epsilon_0 K V}{d}$$

$$\frac{q}{V} = \frac{A\epsilon_0 K}{d} \qquad \frac{q}{V} = C \quad \text{so} \quad \boxed{C = \frac{A\epsilon_0 K}{d}}$$

↑ $C \propto K$

Energy storage in capacitor

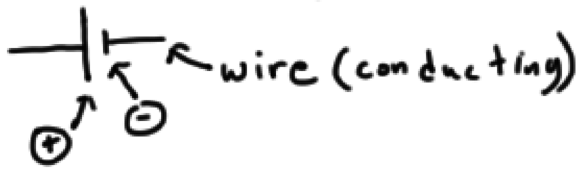
Just remember concept that work must be done to separate charges.

Chapter 20

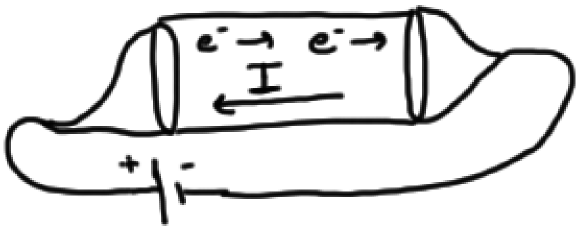
Electromotive force & current

chemical reaction does work to separate charges → get electrical potential difference

We draw a battery in a circuit:



whats happening in a wire?



Take a car battery. Chem. reaction keeps Positive terminal at 12 Volts higher than potential of negative terminal.

Volt → $\frac{\text{Jouls}}{\text{Coulomb}}$

Like $\frac{EPE}{q_0}$

The electromotive force (emf.) is 12 V

In a wire the emf moves the charges.
The flow of charges is called electric current

$$I = \frac{\Delta q}{\Delta t}$$

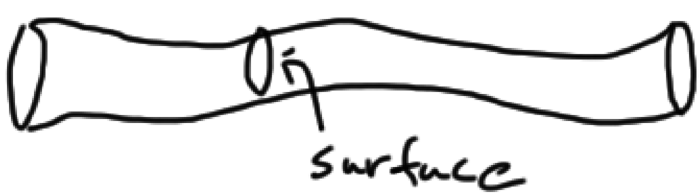
Δq ← amount of charge

Δt ← amount of time

I: elec. current

I has dimensions $\frac{C}{t}$ or coulombs per unit time

To find Δq draw a surface across the complete cross section of the wire.



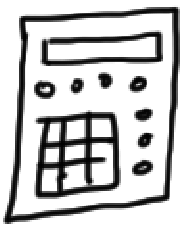
→ Count how much charge (Δq) passes surface in some amount of time (Δt)

$I \rightarrow$ dim. is $\frac{C}{S}$ $\frac{\text{(coulombs)}}{\text{(sec)}}$ $\rightarrow$ called <u>Ampere</u> (A)

If $I(t) = \text{const} \rightarrow$ "Direct Current" DC

If $I(t) \rightarrow$ periodic in t "alternating current" AC

<u>Example problem</u> from book
(using #s!)



$\leftarrow$ 3V Battery

measured current in wire to be 0.17 mA

In one hour a) how much charge flows

b) how much energy expended

a)

$$1\cancel{hr} \times \frac{60\cancel{min}}{\cancel{hr}} \times \frac{60\,sec}{\cancel{min}} = 3{,}600\,s$$

use $I = \frac{\Delta q}{\Delta t}$ $\leftarrow$ looking for amount of charge $\qquad \Delta q = (\Delta t) I$

$I = .17\,mA = .17 \times 10^{-3} A \qquad \Delta t = 3{,}600$

$$\Delta q = 3600_s \times .17 \times 10^{-3} A = \boxed{0.61 C}$$

$\rightarrow$ how many electrons is this?

$e = 1.6 \times 10^{-19} C \qquad$ so $\qquad N_e = \frac{0.61 C}{1.6 \times 10^{-19} C} = \boxed{3.8 \times 10^{18}}$

about the # of atoms in a grain of sand

b) units of volts $= \frac{J}{C}$

so $V \times C = J$

$3V \times .61 C = 1.8 J$ $\leftarrow$ one 300^{th} of <u>firecracker</u>